Optimal control of pruning leaves based on GreenLab

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Abstract

Pruning is a common action when people cultivate crops or trees, in order to make plants grow well and to reach their own aims. Once pruning is done during plant growth, plant growth behavior does have been influenced, as plant architecture is changed due to pruning. Thus, plant yield is variable. The number of leaves is crucial for plant yield as they are the organs that produce plant biomass through photosynthesis. However, one of the most special cases is to prune leaves in agriculture, as leaves are economical valuable, tea plant for instance. Inappropriate pruning of leaves will reduce plant yield and thus result in the loss of the yield of pruned leaves. In order to obtain maximal yield of pruned leaves, a single objective optimal control problem was first formulated based on the functional-structural plant growth model GreenLab (de Reffye et al., 2008) in this paper. The control variable of the problem is pruning strategy, representing the proportion of number of pruned leaves with certain age at certain time among total number of leaves with the age at the time; and the state variable is the total mass of leaves of the corresponding age at the time, which can be derived from the GreenLab model. Due to the mathematical formulism of GreenLab as discrete dynamic system, the formulated optimal control problem can be considered as a discrete dynamic system. The variational method adapted to discrete dynamic system and Lagrange theory were used to calculate the gradient of the objective function of the optimal control problem with respect to the control variable and to solve the problem. Besides the benefit from the yield of pruned leaves, the final benefit should minus the cost of the pruning operation. Hence, a multi-objective optimization problem of pruning strategy was then formulated in this paper. The objectives are maximization of the yield of pruned leaves and minimization of the frequency of pruning operation simultaneously. The mixture algorithm of Particle Swarm Optimization (PSO) (Qi et al., 2010) was used to solve the multi-objective optimization problem. The optimal results of the pruning strategy for both single- and multi-objective optimization problems revealed the optimal trade-off between source-sink dynamics during the plant growth. Moreover, the optimal solutions of the multi-objective optimization problem provided the decisions to decision-makers according to their requirements. It makes GreenLab possible to be used for decision support in agriculture or forestry.

de Reffye, P., Heuvelink, E., Barthélémy, D., and Cournède, P.H. 2008. Plant growth models. In S. Jorgensen and B. Fath, editors, Ecological Models. Vol. 4 of Encyclopedia of Ecology (5 volumes), Elsevier (Oxford), pp. 2824 -2837.

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Introduction

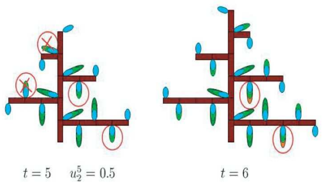
Pruning is a common action when people cultivate crops or trees, in order to make plants grow well and to reach their own aims. Once pruning is done during plant growth, plant growth behavior does have been influenced, as plant architecture is changed due to pruning. Thus, plant yield is variable. The number of leaves is crucial for plant yield as they are the organs that produce plant biomass through photosynthesis. However, one of the most special cases is to prune leaves in agriculture, as leaves are economical valuable, tea plant for instance. Inappropriate pruning of leaves will reduce plant yield and thus result in the loss of the yield of pruned leaves. In order to obtain maximal yield of pruned leaves, a single- and a multi-objective optimal control problems were formulated and investigated based on the functional-structural plant growth model GreenLab.

Problem statement --- optimal control problem

State variable $X(t)$ representing leaf biomass:

$$\begin{cases} X(t) = [x_1(t), x_2(t), \cdots, x_{ta}(t)]^T, 1 \le t \le T_N \\ X(0) = 0 \end{cases}$$

$x_i(t)$: total biomass of leaves of age i at time t;
T_N: final age of the plant;
ta: functioning time of leaves.



Control variable $U(t)$ representing pruning strategy:

$$\begin{cases} U(t) = [u_1(t), u_2(t), \cdots, u_{ta}(t)]^T, 1 \le t \le T_N \\ U(0) = 0 \end{cases}$$

$u_i(t)$: proportion of number of pruned leaves with age i at time t among total number of leaves with age i at time t, $0 \le u_i(t) \le 1$

The yield of pruned leaves at time t is $y(t) = U(t)^T X(t)$

The total yield of pruned leaves at the final plant age is

$$J = \sum_{t=1}^{T_N} y(t) = \sum_{t=1}^{T_N} U(t)^T X(t) = \sum_{t=1}^{T_N} g^t(X(t), U(t))$$ → **objective function**

Method --- GreenLab

functional-structural plant growth model, combining organogenesis and physiological processes(de Reffye et al. 2008).

According to the GreenLab principles, the state variable $x_i(t)$ is given by

$$x_i(t+1) = Nb(i,t+1) \cdot x_{i-1}(t) / Nb(i-1,t) + Nb(i,t+1) p^b(i) \cdot Q(t) / D(t), \forall t \in [0, T_N]$$

$$Nb(i,t) = N^b(t-i+1) \prod_{k=1}^{i-1} (1 - u_{i-k}(t-k)), \forall t \in [1, T_N+1]$$ → $X(t+1) = F^t(X(t), U(t))$

Plant biomass Q by photosynthesis: $Q(t) = \beta E(t)\left(1 - \exp\left(-\gamma S(t)\right)\right)$

Leaf surface area S: $S(t) = \sum_{i=1}^{ta} Nb(i,t) \sum_{j=0}^{i-1} p^a(j) \cdot Q(t-(i-j)) / (D(t-(i-j))e)$

Plant demand D: $D(t) = \sum_o \sum_{j=0}^{tx-1} p^o(j) N^o(t-j+1) + P^r \cdot Q(t) / D(t) \cdot \sum_{k=1}^{ta} Nb(k, t-1)$

$Nb(i,t)$: number of leaves with age i at time t;
γ, β: empirical parameters; e: specific leaf biomass;
$E(t)$: environmental factor at time t;
$N^o(t)$: number of organs (blades a; etc) appeared at time t;
p^o: organ sink function; P^r: combial sink strength;
tx: expansion duration.

Method --- Variational method and Lagrange theory

Converting the maximization problem J to minimization problem J' by multiplying -1, the formula of the optimal control problem is given by

$$\begin{cases} J' = \sum_{t=1}^{T_N} G^t(X(t), U(t)) = \sum_{t=1}^{T_N} -g^t(X(t), U(t)) \\ X(t+1) = F^t(X(t), U(t)), \quad 0 \le t \le T_N \end{cases}$$

Using the variational method and Lagrange theory, the gradient of the objective function J' with respect to U is given by

$$\nabla_U J'(U) = G_U^t(X(t), U(t)) + \lambda^{t+1^T} F_U^t(X(t), U(t)), \quad \forall t \in [1, T_N - 1]$$

$$\begin{cases} \lambda^t = G_X^t(X(t), U(t)) + \lambda^{t+1T} F_X^t(X(t), U(t)), \quad \forall t \in [2, T_N - 1] \\ \lambda^{T_N} = 0 \end{cases}$$

Control plant

The virtual plant we optimized has one order branch and the maximum number of growth units on the branches is 20. The plant age is 70 years. The functioning time and the expansion time of organs are 12 years and 3 years, respectively. The internodes have rings round piths.

Result --- single objective problem

Objective of the problem:
maximize yield of pruned leaves.

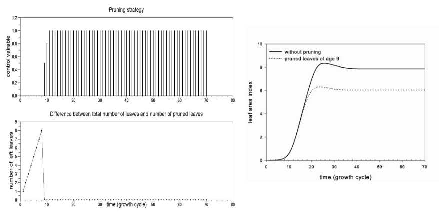
The objective function is given by

$$J = \sum_{t=1}^{T_N} y(t) = \sum_{t=1}^{T_N} U(t)^T X(t) = \sum_{t=1}^{T_N} g^t(X(t), U(t))$$

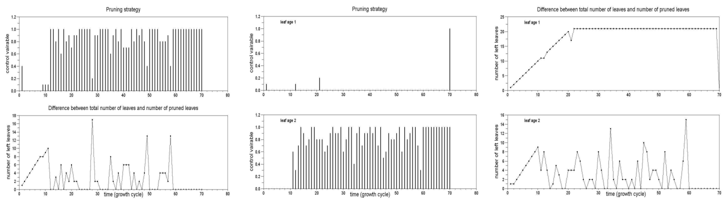
$$\text{subject to } U(t) \in [0,1], \forall t \in [0, T_N]$$
$$u_i(t) = 0, \text{ if } i \ne t_l$$

t_l: age of leaves to be pruned.

The optimized variable is $U(t)$.



Optimal strategy of pruning leaves of age 9, the optimal yield of pruned leaves being 16010.8 g.



Optimal strategy of pruning leaves of age1, the optimal yield of pruned leaves being 8879.8 g.

Optimal strategy of pruning leaves of age 1 and age 2, the optimal yield of pruned leaves being 12940 g.

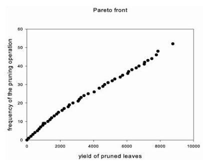
Result --- multi-objective problem

Objective of the problem:
1. maximization of the yield of pruned leaves;
2. minimization of pruning frequency.

The objective function is given by

$$U^* = \arg\max_U Z = \left(\sum_{t=1}^{T_N} U(t)^T X(t), -\sum_{t=1}^{T_N} (\max(u_i(t), i = 1, 2, \ldots, ta))\right)$$

$$\text{subject to } U(t) = 0 \text{ or } 1, u_i(t) = 0, \text{ if } i \notin [t_l, t_r], u_i(t) \in U(t)$$

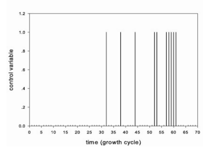


Optimal solution of the problem, known as *Pareto front*, the age of pruned leaves being 1.

$[t_l, t_r]$ is the age range of leaves to be pruned.

The optimized variable is $U(t)$.

Particle Swarm Optimization (PSO) is used to solve the problem (Qi et al. 2010).



Optimal yield is 1255 g.

Optimal pruning strategy corresponding to the *Pareto front*, when the pruning operation is applied ten times, the age of pruned leaves being 1.

Conclusion

1. The potential application of GreenLab for decision support in agriculture or forestry by applying the classical gradient based method and PSO to solve single and multi-objective optimization problems was illustrated.
2. The results of the optimal pruning strategy revealed the optimal trade-off between source-sink dynamics during the plant growth.
3. The optimal solutions of the multi-objective optimization problem provide the decisions to decision-makers according to their requirements.

References

de Reffye, P., Heuvelink, E., Barthélémy, D., and Cournède, P.H. 2008. Plant growth models. In S. Jorgensen and B. Fath, editors, Ecological Models. Vol. 4 of Encyclopedia of Ecology (5 volumes), Elsevier (Oxford), pp. 2824 -2837.
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