





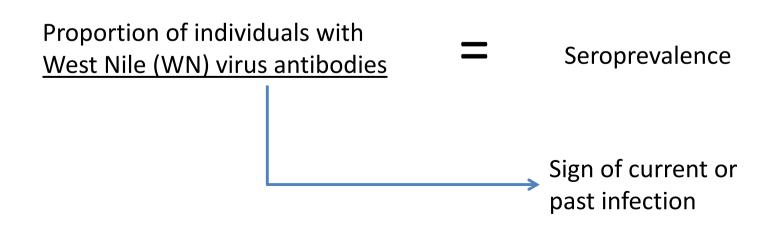
# Generalized linear models (GLM)

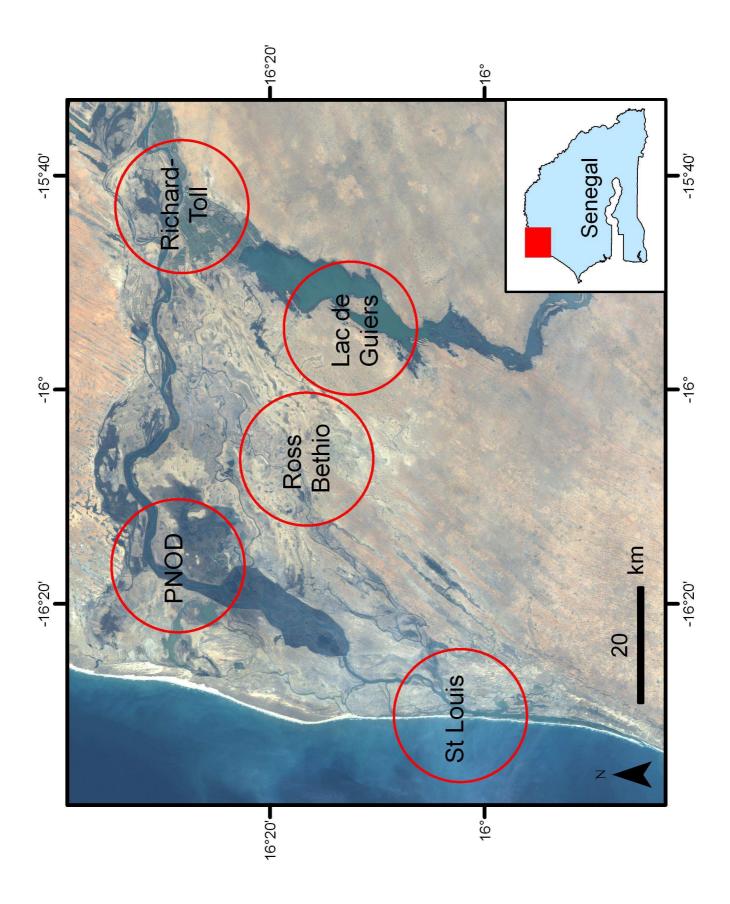
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> CIRAD UR AGIRs

### GLM: Application situation

- 2 categories of individuals in a population
- The variable we wish to model is the proportion of one of these two types
- Example: population of horses in Senegal





#### The data

One sample of horses from that population (Senegal). On each sampled horse, blood sample has been taken and WN antibodies have been searched for

#### Read the data

prevchev<-read.table ("prechev.csv",header=TRUE,sep=",",dec=".")</pre>

#### Look the data

summary(prevchev)

AGE	REGION	SALINITE	VILLAGE	POS	TOT
Min. : 2.000	DJ0:32	Min. :-0.170000	Nguith : 13	Min. : 0.000	Min. : 1.000
1st Qu.: 6.000	NGT:38	1st Qu.:-0.150000	Ross-bethio : 13	1st Qu.: 1.000	1st Qu.: 1.000
Median : 8.000	RIT:43	Median :-0.090000	Tiguette : 13	Median : 1.000	Median : 1.000
Mean : 8.611	ROB:46	Mean :-0.005657	Débi : 12	Mean : 1.586	Mean : 1.854
3rd Qu.:10.000	STL:39	3rd Qu.:-0.040000	Gohou Mbathie: 12	3rd Qu.: 2.000	3rd Qu.: 2.000
Max. :24.000		Max. : 0.450000	Mbodiene : 9	Max. :12.000	Max. :13.000
			(Other) :126		
			$\mathbf{N}$		
			X		
			132 v	illages	

#### Data presentation

#### Look at the first 10 lines

#### head(prevchev,10)

	AGE	REGION	SALINITE	V	LLAGE	POS	TOT
1	6	NGT	-0.15	Belel	mbaye	1	1
2	11	NGT	-0.15	Belli	bamdi	1	1
3	4	ROB	-0.17	Bisse	ette 1	1	1
4	10	RIT	-0.09	Camp	pement	1	1
5	7	NGT	-0.15	Darou	salam	1	1
6	8	NGT	-0.15	Darou	salam	1	1
7	10	NGT	-0.15	Darou	salam	2	2
8	2	DJO	-0.04		Débi	2	4
9	3	DJO	-0.04		Débi	2	3
10	4	DJO	-0.04		Débi	4	8

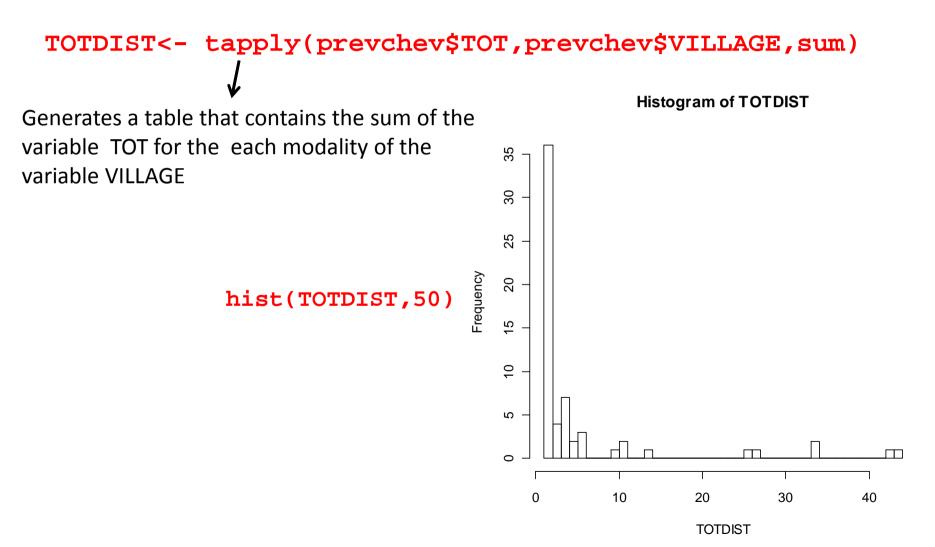
#### **Dimensions of the data table**

dim(prevchev)

198 6 → 6 variables
 Q
 One statistical unit per age\*village class(132 villages)

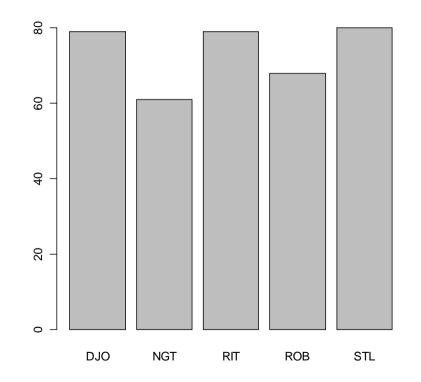
#### Description of the data: number of individuals sampled

Distribution of the number of horses sampled in a village ! Sometimes, more than one stat unit in a village (age classes)



# Distribution of the number of horses sampled in each region

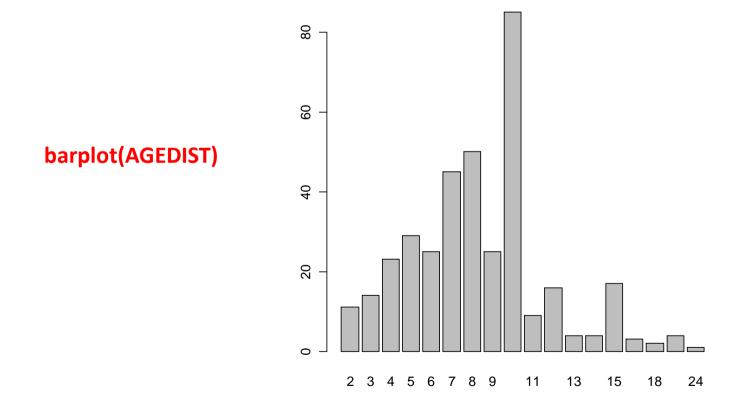
REGDISTCHEV<-tapply(prevchev\$TOT,prevchev\$REGION,sum) barplot(REGDISTCHEV)



#### Description of the data: distribution / age

Distribution of the age of sampled horses

AGEDIST<- tapply(prevchev\$TOT,prevchev\$AGE,sum)



### Aims of the study

- Estimate the proportion of horses with antibodies
- Determine the influence on the response variable (proportion of seropositive horses) of the explanatory variables:
  - Age
  - Region
  - Salinity

# Type of model depend on types of response and explanatory variables

Response	Explanatory	Statistical model	
Continuous	All continuous	Linear regression	
Continuous	All categorical	Analysis of variance (ANOVA)	
Continuous	Continuous and categorical	Analysis of covariance (ANCOVA)	
Continuous	Any combination of continuous and/or categorical variables	Linear model (LM)	
Categorical, Count, Probability, Proportion	Any combination of continuous and/or categorical variables	Generalized linear models (GLM)	

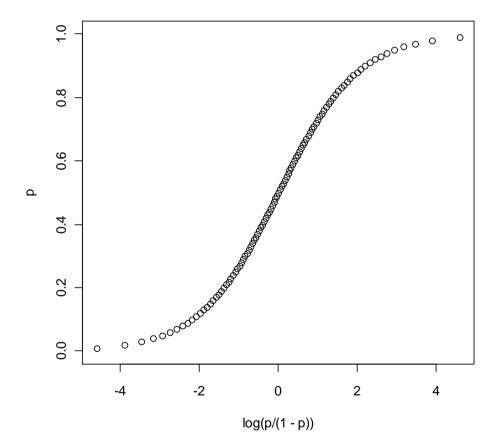
### What type of model shall we use?

- A generalized linear model (GLM) characterized by :
  - A link function
  - A distribution law

### Which link function ?

- The logit function
  - p is outcome proportion
  - Logit(p) =  $\log(p/(1-p))$

p=seq(0,1,0.01)
plot(log(p/(1-p)),p)



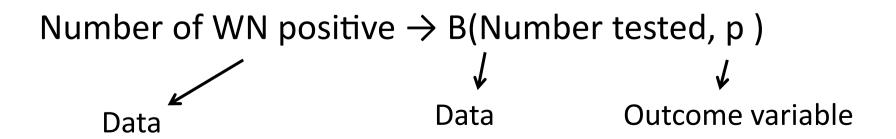
#### Interpretation of logit

- Logit(p) = log(p/(1-p))
  - Logit is the log of odd
  - Way of expressing probabilities originating from gambling vocabulary
  - A horse with an odd of 25 against 1 = 25 times more likely to loose the race than to win it
  - Scientists use p, gamblers p/(1-p)
  - With the logit function think in terms of log(p/(1-p))

### What underlying distribution ?

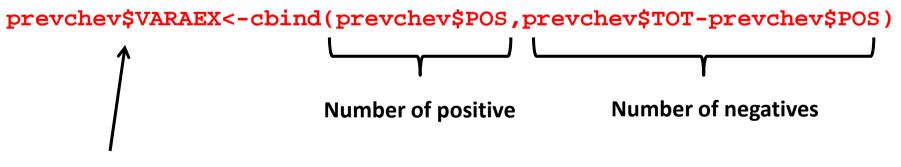
### • The binomial distribution

- Classical example : numbers of 3s for 6 dice draws: B(6, 1/6)
- Describe a number of events given:
  - The probability of the event
  - The number of trials (draws)
- For modelling a proportion:
  - The data include the number of events
  - The data include the number of trials
  - We want to estimate the probability of an event



### Outcome variable for a proportion

## Outcome variable has 2 components: •number of positive •number of négatifs



The outcome variable

# Model syntax in R

In R the model is usually defined with a function including a formula as one argument

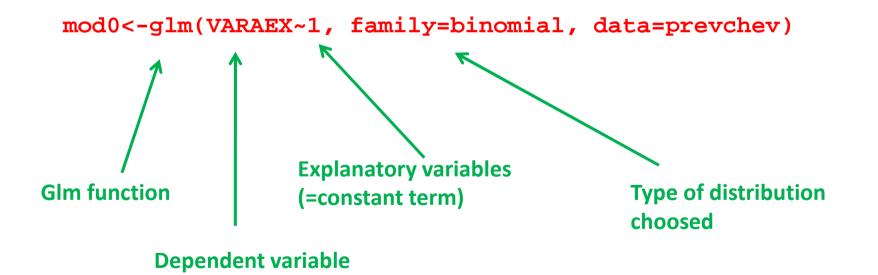
aov(formula,options)
lm(formula,options)
glm(formula,options)

Analysis of variance and covariance (ANOVA, ANCOVA) Regressions and linear models Generalized linear models

#### Formula syntax

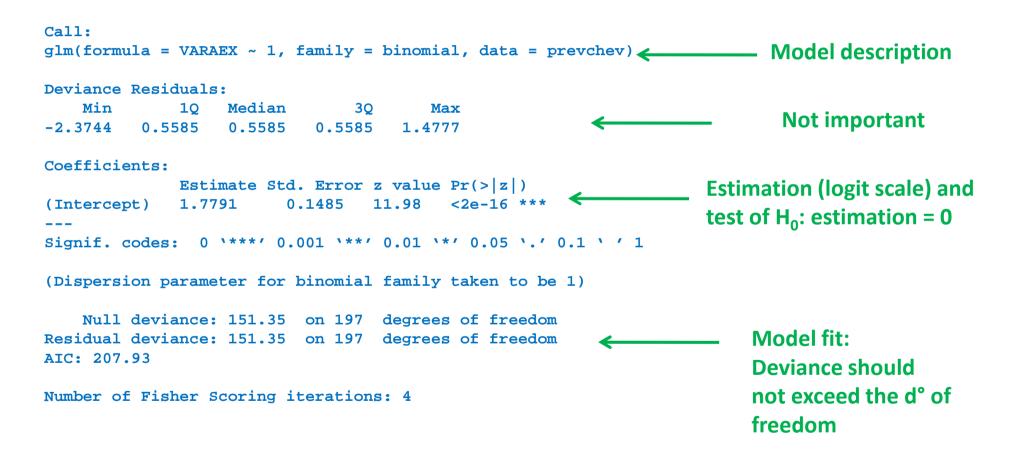
~	separates response and explanatory variables
+	addition of an explanatory variable and b
a:b	interaction between a and b
a*b	equivalent to a+b+a:b
a/b	b is nest in a

The null model is the simplest one can build: it considers the proportion as homogeneous



#### Interpretation of the outputs

#### summary(mod0)



### How is the model fitted ?

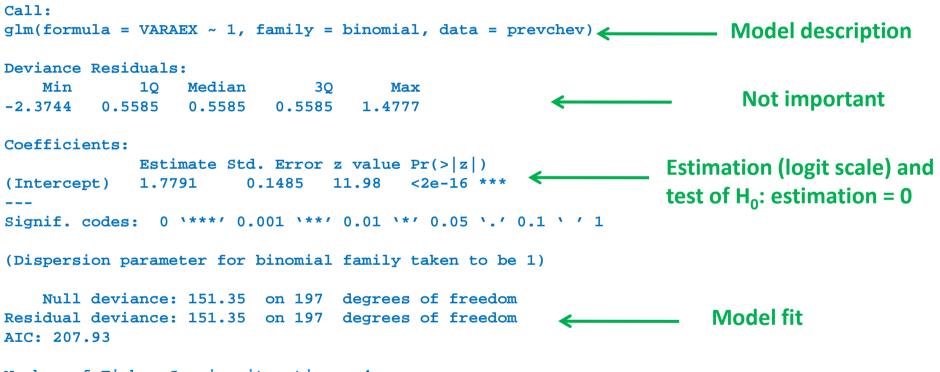
### Maximum likelihood method:

•Determine the value of the parameter that maximises the probability of the data

•Given the structure of the model (*i.e.* considering that the proportion WN positive individuals is homogeneous)

#### Interpretation of the outputs

#### summary(mod0)



Number of Fisher Scoring iterations: 4

### The estimation obtained is 1.7791

•It is the logit of the estimation of the proportion of WN positive individuals in a village under the hypothesis that this proportion is homogenous

•To obtain the proportion estimation, one has to apply the inverse logit function

```
\exp(1.7791)/(1+\exp(1.7791))
```

```
0.8555857
```

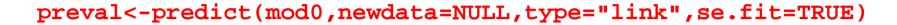
computes the value predicted by the fit<-fitted.values(mod0) or head(fit,5) the data table

model for each statistical unit (line) in

```
1
                2
                            3
                                        4
                                                     5
   /0.8555858 0.8555858 0.8555858 0.8555858 0.8555858
Directly in proportion and not
any more in Logit
```

What about the confidence interval !!!!!!!

### Estimation of logit(p) with confidence interval



Name of the model used for computing the estimations

Table listing the combinations of the explanatory variables for which estimations are required. If NULL, the data table is used Scale of estimationEstimation logi(p): « link » or p: « response »

Erreurs standards des estimations

#### Generates a 3 components list: \$fit, \$se.fit, \$residual scale

head(preval\$fit,5)
1.779101 1.779101 1.779101 1.779101

#### head(preval\$se.fit,5)

0.1485006 0.1485006 0.1485006 0.1485006 0.1485006

logit (p) is estimated at 1.7791 with a standard error of 0.1485

#### Estimation of p with confidence interval

logit (p) is estimated at 1.7791 with a standard error of 0.1485

The 95% confidence interval of logit(p) can be build
Lower limit: 1.7791-1.96\*0.1485
Upper limit: 1.7791+1.96\*0.1485
2.07

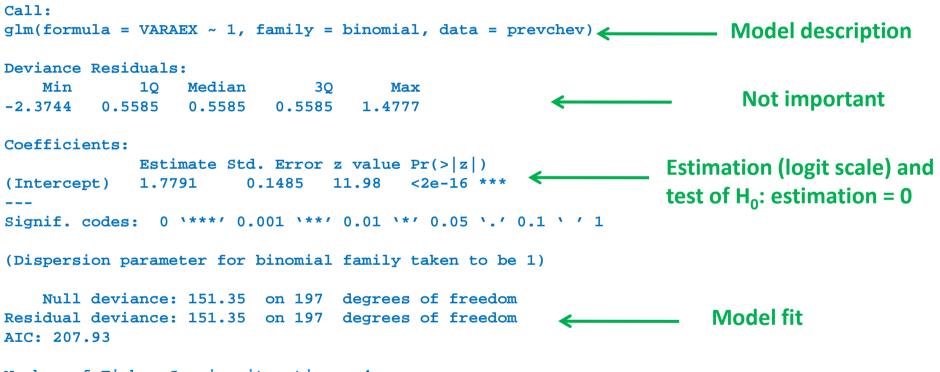
To compute the confidence interval of the estimation of p

The inverse logit function is applied to the limits of the logit(p) IC
Estimation of p: exp(1.7791)/(1+exp(1.7791)) 0.856
lower limit: exp(1.488)/(1+exp(1.488)) 0.816
upper limit: exp(2.07)/(1+exp(2.07)) 0.888

p is estimated at 0.856 with a 95% IC = [0.816; 0.888]

#### Interpretation of the outputs

#### summary(mod0)



Number of Fisher Scoring iterations: 4

Null deviance:151.35on 197degrees of freedomResidual deviance:151.35on 197degrees of freedomAIC:207.93

The deviance: Quantity of variation in the data unexplained by the model

•The larger is the deviance, the larger is the quantity of unexplained variation

•Null deviance: the deviance of null model (the model in which the response variable is considered as homogeneous)

•Residual deviance: deviance of the current model (note that here null deviance = residual deviance because the current model is the null model)

•The degrees of freedom = number of statistical units –number of parameters in the model

**Model fit:** residual deviance ≈ number of degrees of freedom

If residual deviance >> residual ddl
 The model doesn't contain any important explanatory variable
 La chosen distribution (binomial) is not adapted

Null deviance:151.35on 197degrees of freedomResidual deviance:151.35on 197degrees of freedomAIC:207.93

AIC: is a measure of model quality in terms of quantity of explained variation and parameter number

•For a given deviance, AIC selects the model with the lower number of parameters

•For a given number of parameters, AIC selects the model of lowest deviance

•The smaller the AIC, the best is the model

•A difference of 2 AIC points between 2 models is significant (the model with the lowest AIC is significantly better)

A model to test the effect of a continuous explanatory variable : age

### Now we add the effect of age in the model

```
mod1<-glm(VARAEX~1+AGE, family=binomial, data=prevchev)</pre>
Or
    mod1<-update(mod0,~.+AGE)</pre>
  Call:
  glm(formula = VARAEX ~ AGE, family = binomial, data = prevchev)
  Deviance Residuals:
                                                                  Estimation of the effect
                10 Median
      Min
                                 30
                                         Max
  -2.8685 0.2642
                    0.4970
                             0.6448
                                      1.5660
                                                                  of age. The proportion of
                                                                  WN positives increases
  Coefficients:
              Estimate Std. Error z value Pr(>|z|)
                                                                  with age.
  (Intercept) 0.33691
                         0.39309 0.857 0.391390
                                   3.647 0.000265 ***
                         0.05155
  AGE
               0.18804
  _ _ _ _
                 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
  Signif. codes:
  (Dispersion parameter for binomial family taken to be 1)
                                                                     The addition of the age
      Null deviance: 151.35 on 197 degrees of freedom
  Residual deviance: 136.18 on 196 degrees of freedom
                                                                     affect results in a
  AIC: 194.75
                                                                     decrease of the deviance
```

#### Test of the effect of age

•Z-test on the coefficient: test of H<sub>0</sub>: coef(AGE)=0

```
Coefficients:

Estimate Std. Error z value Pr(>|z|)

(Intercept) 0.33691 0.39309 0.857 0.391390 P-value <0.05.

AGE 0.18804 0.05155 3.647 0.000265 *** Significant age effect

----

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1
```

Test of the likelihood ratio between mod0 and mod1

The deviance difference between two nested models ~ a  $\chi^2$  distribution with nb of df = difference between the residual degrees of freedom of the two models

```
deviance(mod0)-deviance(mod1) 15.17363
df.residual(mod0)-df.residual(mod1) 1
1-pchisq(15.7,1) 9.825205e-05 P-value <0.05
Significant age effect</pre>
```

•Comparaison of the AIC of mod1 and mod0

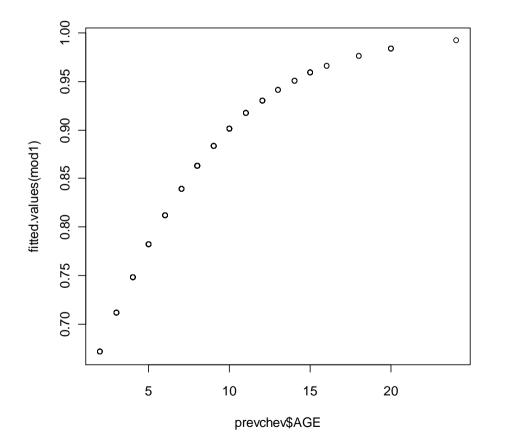
AIC(mod1)	194.75	The model with age has a lower AIC. The age offect is
AIC(mod0)	207.93	The model with age has a lower AIC. The age effect is significant

#### Representation of the age effect

plot(prevchev\$AGE, fitted.values(mod1))



Values predicted by the model including the effect of age

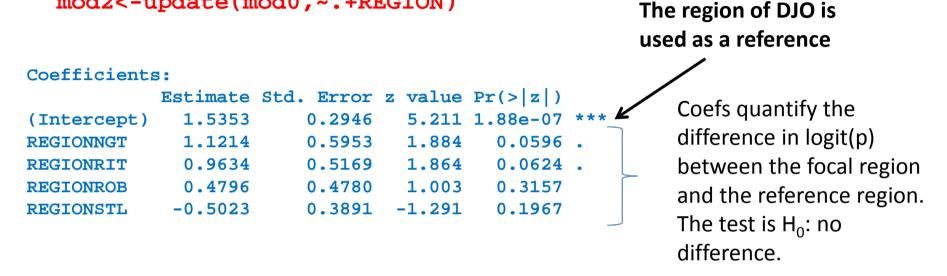


•Non-linearity: the relationship is linear on the logit scale but not on the proportion scale

•With the logit link, the predicted values are not above 1

#### A model to assess the effect of a categorical variable : region

```
mod2<-glm(VARAEX~1+REGION, family=binomial, data=prevchev)
Or
mod2<-update(mod0,~.+REGION)
The region of DIO is</pre>
```



mod2<-glm(VARAEX~REGION-1, family=binomial, data=prevchev)
Or mod2<-update(mod0,~.-1+REGION)</pre>

	Estimate	Std. Error	z value Pr(> z )
REGIONDJO	1.5353	0.2946	5.211 1.88e-07 ***
REGIONNGT	2.6568	0.5172	5.136 2.80e-07 ***
REGIONRIT	2.4987	0.4247	5.884 4.02e-09 ***
REGIONROB	2.0149	0.3764	5.353 8.64e-08 ***
REGIONSTL	1.0330	0.2541	4.065 4.80e-05 ***

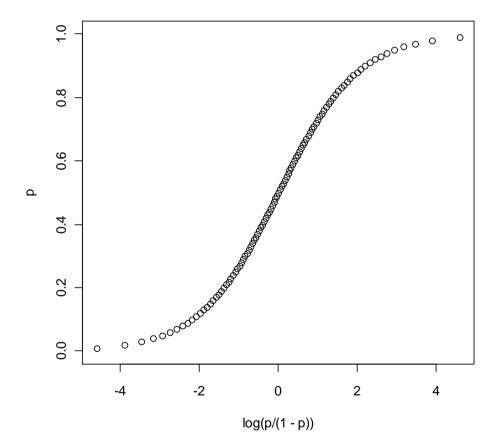
Coefs give logit(p) for the focal region. Test  $\underline{H_0}$ : logit(p)= 0 it means p=0.5

**Usually not interesting** 

### Which link function ?

- The logit function
  - p is outcome proportion
  - Logit(p) =  $\log(p/(1-p))$

p=seq(0,1,0.01)
plot(log(p/(1-p)),p)



#### Test of the region effect

Z-test on the coefficient: not very usefull
Either a test of the difference with an arbitrarily determined reference region
Or a test of H<sub>0</sub> p=0.5

#### •Test of the likelihood ratio between mod0 and mod2

The deviance difference between two nested models ~ a  $\chi^2$  distribution with nb of df = difference between the residual degrees of freedom of the two models

<pre>deviance(mod0)-deviance(mod2)</pre>	15.89853	
df.residual(mod0)-df.residual(mod2)	4	
1-pchisg(15.8985,4)	0.003	P-value <0.05
_ pomp4(10.0000/1)	0.005	Significant region

#### •Comparaison of the AIC of mod2 and mod0

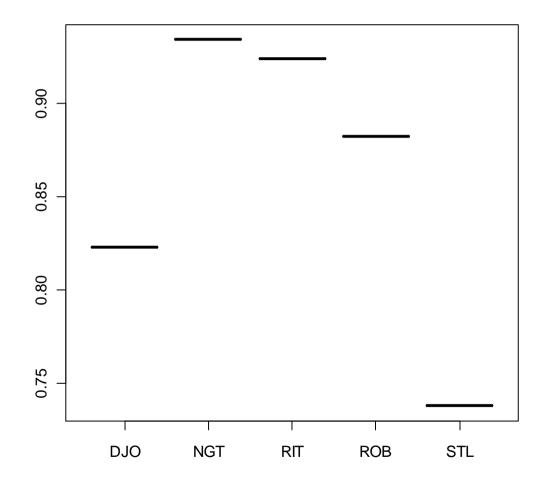
AIC(mod1)	200.03
AIC(mod0)	207.93

AIC of the model including region is smaller. Region has a significant effect on the proportion of WN positive

effect

#### Representation of the region effect

plot(previnddjo\$REGION, fitted.values(mod2))



We want to identify the model built with the available explanatory variables that provides the best possible description of the data

## Modelling principles

Model	Interpretation
Saturated model	<ul> <li>Includes one parameter by data point.</li> <li>Describe perfectly the data but is useless for inferring the mechanisms that generate variation in the response variable.</li> </ul>
Maximal model	<ul> <li>Includes the effects of all the potential explanatory variables and all their interactions.</li> <li>Usually used as a starting point for the model selection process</li> </ul>
Minimal adequate model	<ul> <li>Includes only the effects of the potential variables and of the interactions which removal results in a significant decrease in the fraction of explained variation</li> <li>The description of the response variable retained</li> </ul>
Null model	<ul> <li>Includes only one parameter which represent the estimation of the response variable under the hypothesis that it is homogeneous in the population (no variation).</li> <li>A kind of baseline model: models that do not explain more variation can be considered as irrelevant.</li> </ul>

We want to identify the model built with the available explanatory variable that provides the best possible description of the data

If we consider only AGE and REGION as potential explanatory variables,

The maximal model contains the effect of

•AGE

•REGION

And the interaction AGE\*REGION

The effect of age differs among regions

## Modelling: building the maximal model

### Maximal model

mod3<-glm(VARAEX~REGION+AGE+AGE\*REGION, family=binomial, data=prevchev)</pre>

#### Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-0.37072	0.67514	-0.549	0.58294	Reference coefficient
AGE	0.29696	0.11079	2.680	0.00735	<b>**</b> Age coefficient
REGIONNGT	0.11548	1.57426	0.073	0.94152	
REGIONRIT	1.91226	1.47469	1.297	0.19473	Design Coefficients
REGIONROB	0.74785	1.12760	0.663	0.50719	Region Coefficients
REGIONSTL	1.17632	1.12666	1.044	0.29645	
AGE: REGIONNGT	0.10675	0.24481	0.436	0.66280	1
AGE:REGIONRIT	-0.17889	0.19490	-0.918	0.35871	Internetion Coofficients
AGE: REGIONROB	-0.08362	0.16524	-0.506	0.61283	Interaction Coefficients
AGE:REGIONSTL	-0.26914	0.15357	-1.753	0.07968	•
Signif. codes	: 0 `***	′ 0.001 `**′	0.01	*′ 0.05 `.	' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 151.35 on 197 degrees of freedom Residual deviance: 116.52 on 188 degrees of freedom AIC: 191.1

Number of Fisher Scoring iterations: 6

### Modelling: reaching the minimum adequate model

We start from the maximal model and remove non significant effects

One can use Likelihood Ratio Tests to remove the non significant effects
Start by trying to remove the interactions
Do not remove a main effect when it is involved in an interaction

•One can use an automatic removal procedure based on AIC comparisons

```
library(MASS)
stepAIC(mod3)
Start: AIC=191.1
VARAEX ~ AGE + REGION + AGE * REGION
             Df Deviance
                           AIC
- AGE: REGION 4 121.17 187.75
                 116.53 191.10
<none>
Step: AIC=187.75
VARAEX ~ AGE + REGION
         Df Deviance
                       ATC
       121.17 187.75
<none>
- REGION 4 136.18 194.75
- AGE 1 135.45 200.03
```

The minimum adequate model includes the effects of AGE and REGION but not their interaction

## Minimum adequate model = final model

modfin<-glm(VARAEX~AGE+REGION, family=binomial, data=prevchev)</pre>

Coefficients:						
COETTCIENC		_	_			
	Estimate St	d. Error z	value	Pr(> z )		
(Intercept)	0.22085	0.44961	0.491	0.623276		
AGE	0.19282	0.05513	3.498	0.000469	* * *	
REGIONNGT	0.91045	0.60840	1.496	0.134534		
REGIONRIT	0.75867	0.53074	1.429	0.152871		
REGIONROB	0.29442	0.49332	0.597	0.550636		
REGIONSTL	-0.72290	0.41095	-1.759	0.078560	•	
Signif. cod	es: 0 `***'	0.001 `**	• 0.01	<b>`*'</b> 0.05	`.' 0.1	<b>` ′ 1</b>
(Dispersion	parameter f	or binomia	al famil	y taken t	co be 1)	
Null deviance: 151.35 on 197 degrees of freedom						

Residual deviance: 121.17 on 192 degrees of freedom AIC: 187.75

## Obtaining the estimations from the final model

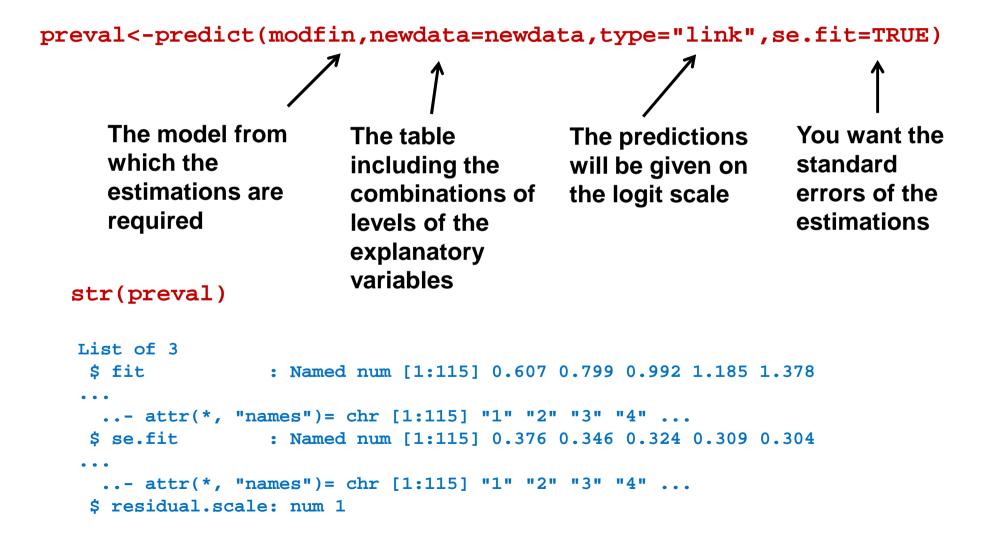
Create a data frame including the combinations of levels of the explanatory variables for which we want to get an estimations of prevalence

```
newdata<-
expand.grid(AGE=seq(2,24,1),REGION=levels(prevchev$REGION))</pre>
```

#### newdata

	AGE	REGION
1	2	DJO
2	3	DJO
3	4	DJO
4	5	DJO
5	6	DJO
6	7	DJO
7	8	DJO
8	9	DJO
9	10	DJO
10	11	DJO
11	12	DJO
12	•••••	

•Use the predict() function to obtain estimations from the final model for the combinations of levels of the explanatory variables



### •Add the predictions and confidence intervals in the newdata table

```
newdata$pred<-exp(preval$fit)/(1+exp(preval$fit))
newdata$low<-exp(preval$fit-1.96*preval$se.fit)/(1+exp(preval$fit-1.96*preval$se.fit))
newdata$hig<-exp(preval$fit+1.96*preval$se.fit)/(1+exp(preval$fit+1.96*preval$se.fit))</pre>
```

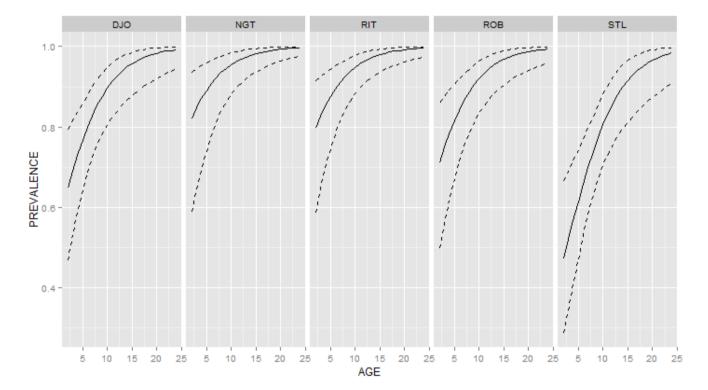
### head(newdata)

Α	GE RE	GION	pred	low	hig
1	2	DJO	0.6471421	0.4674587	0.7930405
2	3	DJO	0.6898298	0.5300237	0.8143319
3	4	DJO	0.7295117	0.5884477	0.8357227
4	5	DJO	0.7658401	0.6407583	0.8570843
5	6	DJO	0.7986362	0.6859485	0.8780763
6	7	DJO	0.8278712	0.7240064	0.8981474

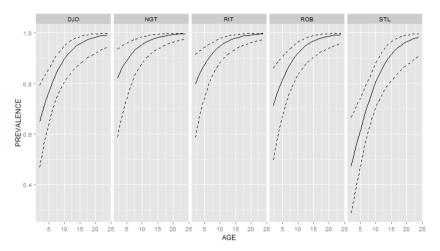
# Graphic

### (http://www.cookbook-r.com/Graphs/)

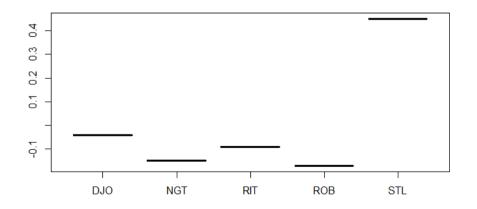
```
ibrary(ggplot2)
ggplot() + geom_line(data=newdata, aes(x=AGE,y=pred)) +
facet_grid(.~REGION)+
geom_line(data=newdata, aes(x=AGE,y=low),linetype=2) +
geom_line(data=newdata, aes(x=AGE,y=hig),linetype=2) +
xlab("AGE") + ylab("PREVALENCE")
```



# Effect of salinity



boxplot(prevchev\$SALINITE~prevchev\$REGION)



Salinity provided at the regional scale, not at the village scale. Lower prevalence in the region with the highest salinity

Create a table including the combinations of the levels of the explanatory variables for which we want estimations from the final model

```
newdata<-as.data.frame(matrix(,nrow=5*23,ncol=2))
names(newdata)<-c("AGE","REGION")</pre>
```

```
newdata$AGE<-rep(seq(2,24,1),5)</pre>
```

```
newdata$REGION<-
c(rep("DJO",23),rep("NGT",23),rep("RIT",23),rep("ROB",23),rep("STL",23))</pre>
```

#### newdata

	AGE	REGION
1	2	DJO
2	3	DJO
3	4	DJO
4	5	DJO
5	6	DJO
6	7	DJO
7	8	DJO
8	9	DJO
9	10	DJO
10	11	DJO
11	12	DJO
12	•••••	

•Use the predict function to obtain estimations model modfin for the combinations of the levels of the explanatory variables listed in newdata

preval<-predict (modfin, newdata=newdata, type="response")

The model used to
obtain the
predictions
The table
containing the
combinaitions of
explanatory
variable levels

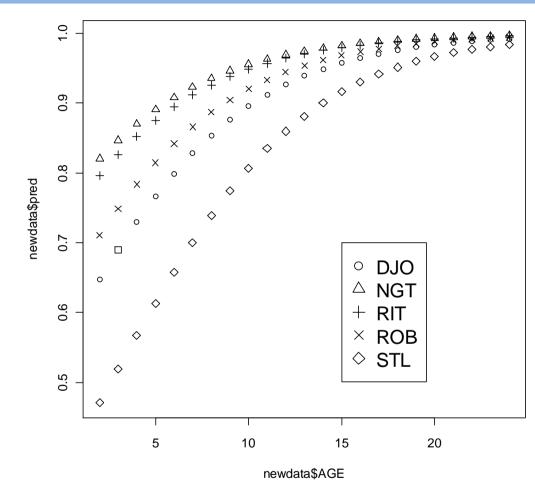
•The predictions are pasted in the newdata table newdata\$pred<-preval

•Crate an numeric equivalent of REGION (1 distinct digit for each region)

```
newdata$regnum<-rep(1,115)
newdata$regnum<-replace(newdata$regnum,newdata$REGION=="NGT",2)
newdata$regnum<-replace(newdata$regnum,newdata$REGION=="RIT",3)
newdata$regnum<-replace(newdata$regnum,newdata$REGION=="ROB",4)
newdata$regnum<-replace(newdata$regnum,newdata$REGION=="STL",5)</pre>
```

### •The predictions (y-axis) are displayed as a function AGE (x-axis) et of REGION (symbol, coded by regnum)

plot(newdata\$AGE,newdata\$pred,pch=newdata\$regnum)
legend(15,0.7,c("DJO","NGT","RIT","ROB","STL"),cex=1.5,pch=1:5)



Additive effects of AGE and REGION

Les predicted lines are parallel on the logit(p) scale, but not on the p scale

## Other GLMs

Type of data	GLM specifications
A number of events in a population of unknown size (number of cases of a	Link Function=log
disease)	Distribution=Poisson
A number of events within a time period of a given length	Be careful for the output interpretation the link function is log.
Size of a group	Otherwise, the same as for a proportion
A binary variable binaire (two possible outcomes) at the individual scale. Each line	Link function=logit Distribution=binomial
is an individual, the dependent variable can only take 2 values (coded 1 or 0)	Be careful: the outcome variable has only one component (not two components as when the outcome variable is a proportion)
	Otherwise, the same as for a proportion

glm(nbre~a+b+a\*b, family=poisson, data=mydata)
glm(bin~a+b+a\*b, family=binomial, data=mydata)