Cellular automata
Individual-based modeling

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Individuals/Environment/Population

Environmental properties
- Climatic and geophysical conditions
  - Space
  - Food
  - Predators
  - Competitors

Individuals’ properties
- Nutrition
- Growth
- Reproduction
- Mobility
- Competitive ability

Demographic processes
- Birthrate
- Mortality
- Emigration
- Immigration

Population state variables
- Density
- Spatial distribution
- Age structure
- Social structure
- Genic frequencies

Feedback

(from Barbault, 1992)
Mathematical models of population dynamics

Mean field models
- The state variable is one number
- Density, frequency, biomass, etc.

Age or stage structured models
- The state variable is a vector (matrix models)
- Density in each age class (Leslie)
  or life-stages (Lefkovitch)

Individual based models
- The state variable is a vector of individual states
Approaches of complex systems

- **Analytical**
  - element by element
  - (neo-classical economy, plot, individual, etc.)

- **Holistic or systemic**
  - global behaviour of the system
  - (macro-economy, statistics)

- **Constructivist**
  - articulation between individual behaviours of the elements (local) and the global behaviour of the system (global)
## Mathematical vs. simulation-based approach

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Analytical</th>
<th>Simulation-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial environment</td>
<td>homogeneous</td>
<td>heterogeneous</td>
</tr>
<tr>
<td>Demographic stochasticity</td>
<td>unimportant</td>
<td>important</td>
</tr>
<tr>
<td>Rare events</td>
<td>unimportant</td>
<td>important</td>
</tr>
<tr>
<td>Biological and/or environmental discontinuities</td>
<td>unimportant</td>
<td>important</td>
</tr>
<tr>
<td>Number of individuals</td>
<td>large</td>
<td>small</td>
</tr>
<tr>
<td>Biological complexity</td>
<td>simple</td>
<td>complex</td>
</tr>
</tbody>
</table>

*(from Gross et al., 1992)*
Spatially-explicit & individual-based simulation of population dynamics

Cellular automata

Individual-based model

Metapopulation model
Cellular automata
Automaton

- A set of inputs
- A set of states
- A transition function
- A set of outputs

<table>
<thead>
<tr>
<th>T</th>
<th>Input</th>
<th>State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nothing</td>
<td>Waiting</td>
<td>Menu</td>
</tr>
<tr>
<td>1</td>
<td>Ask coffee</td>
<td>Waiting</td>
<td>Menu</td>
</tr>
<tr>
<td>2</td>
<td>Nothing</td>
<td>Need = 2</td>
<td>Menu</td>
</tr>
<tr>
<td>3</td>
<td>Nothing</td>
<td>Need = 2</td>
<td>Need 2€</td>
</tr>
<tr>
<td>4</td>
<td>1€ coin</td>
<td>Need = 2</td>
<td>Need 2€</td>
</tr>
<tr>
<td>5</td>
<td>Nothing</td>
<td>Need = 1</td>
<td>Need 2€</td>
</tr>
<tr>
<td>6</td>
<td>Nothing</td>
<td>Need = 1</td>
<td>Need 1€</td>
</tr>
<tr>
<td>7</td>
<td>1€ coin</td>
<td>Need = 1</td>
<td>Need 1€</td>
</tr>
<tr>
<td>8</td>
<td>Nothing</td>
<td>Need = 0</td>
<td>Need 1€</td>
</tr>
<tr>
<td>9</td>
<td>Nothing</td>
<td>Waiting</td>
<td>Coffee</td>
</tr>
<tr>
<td>10</td>
<td>Nothing</td>
<td>Waiting</td>
<td>Menu</td>
</tr>
</tbody>
</table>
Network of automata

- A group of automata, the inputs for some are outputs for others

- Architecture: regular, total connectivity, random, layered
Cellular Automata

- Regular architecture
- Uniform and discrete transition function
- Synchronous and deterministic functioning
In the *Game of Life* proposed by John Conway, cells states (alive -> green; dead -> grey) are changing according to their own state and the states of their 8 neighboring cells.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td></td>
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<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
Look at cell #10 and at its 8 neighboring cells.

As for any alive cell, we need to consider this rule:
(i) it cannot survive to a too wide isolation (less than 2 alive neighbors),
(ii) it will also be killed by a too strong concentration (more than 3 alive neighbors)
Oh! Cell 10 has exactly 3 alive neighbors!

Right, so next step it will still be alive!
Well, birth supposes a certain gathering of population... In the “Game of Life” this has been set to exactly 3 alive neighboring cells...

It is dead! Does it have any chance to become alive?

<table>
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<td>16</td>
</tr>
</tbody>
</table>
Great! Cell 7 has exactly 3 alive neighbors!

Right, so next step it will become alive!

Next step
Summary of rules (transition function)

These rules have to be applied to all cells in the same way to determine the next generation.

- **Alive neighbors < 2** or **Alive neighbors > 3**
- **Alive neighbors = 2** or **Alive neighbors = 3**
- **Alive neighbors ≠ 3**
Individual-based modeling

- water source
- toxic food or water source
- cover
- shade
- dangerous place
- landmark
- cereal type food
- fruit type food
- den
- irrelevant animal (just needs avoiding)

(from Tyrrell, 1993)
Dog rabies
Mathematical approach vs simulation-based approach

Transitions between rabies classes
set of 3 coupled, first-order, non-linear differential equations

\[
\frac{dS}{dt} = aS - S(\beta I + b + \gamma N) \quad (1)
\]

\[
\frac{dL}{dt} = \beta SI - (\sigma + b)L - \gamma NL \quad (2)
\]

\[
\frac{dI}{dt} = \sigma L - (\alpha + b)I - \gamma NI \quad (3)
\]

Anderson et al., 1981
Dog rabies
Mathematical approach
vs simulation-based approach

Density of dogs and persistence of rabies

Basic reproduction of rabies in the dog population
\[ R_0 = \frac{\sigma \beta S}{(\sigma + a)(\alpha + a)} \] (4)

Threshold density of dogs required for rabies transmission
\[ K_{r} = \frac{(\sigma + a)(\alpha + a)}{\sigma \beta} \] (5)

When the density of dogs is below \( K_r \), rabies cannot persist in the population

Vaccination strategies should be related to density of dogs

Kitala et al., 2002
Anderson et al., 1981
Dog rabies dynamics
 Mathematical approach vs simulation-based approach

Vaccination

\[
\begin{align*}
dV/dt &= \varphi S - bV - \gamma NV \quad (5) \\
dS/dt &= a(S + V) - S(\beta I + b + \varphi + \gamma N) \quad (1)
\end{align*}
\]

Comparison of vaccination rates per year
Comparison of yearly vs twice yearly vaccination coverage

Kitala et al., 2002
Dog rabies dynamics
Mathematical approach
vs simulation-based approach

Rates per year (population) -> simulated events (individuals)

\[
\frac{dS}{dt} = a(S + V) - S(\beta I + b + \varphi + \gamma N)
\]
\[
\frac{dL}{dt} = \beta SI - (\sigma + b)L - \gamma NL
\]
\[
\frac{dl}{dt} = \sigma L - (\alpha + b)l - \gamma NI
\]
\[
\frac{dV}{dt} = \varphi S - bV - \gamma NV
\]

Spatial heterogeneity

Types of dogs
Home-owned Community-owned Stray

Vaccination strategies