Modeling the Trade-off between Replication and Transmission in Spore-producing Pathogens

Audrey Andanson [1,2], Fabien Halkett [1], Virginie Ravigné [2]

[1] UMR Interactions Arbres-Microorganismes, INRA-Nancy Université, Route d'Amance, 54280 Champenoux, France

[2] UMR Biologie et Génétique des Interactions des Plantes-Parasites, CIRAD-INRA-Montpellier SupAgro, Campus International de Baillarguet, TA A-54/K, 34398 Montpellier cedex 5, France

Contact: audrey.andanson@cirad.fr

The Spore Production Dilemma

For spore-producing pathogens, within-host replication and transmission are two separate functions consuming energy and hence competing for host limited resources.

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Parasites face a **trade-off** between the benefits of increased within-host replication (*i.e.*, increased instantaneous transmission rate) and the costs (*i.e.*, decreased the amount of resources available for pathogen transmission).

What is the optimal allocation strategy of a finite amount of resources between replication and sporulation functions?

Method

Dynamic optimization (**Pontryagin maximum principle**)

We seek for the shape of the continuous function *u*(*a*) maximizing the fitness of the pathogen (Φ), estimated as its lifetime reproductive success:

$$\Phi = \int_0^{+\infty} u(a) c_2 \varepsilon R M e^{-\mu a} da$$

The Model

Resource Allocation Model describing the within-host dynamics of the pathogen[1] (resource consumption, within-host replication, sporulation).

$$\frac{dR}{da} = -\varepsilon R M$$

$$\frac{dM}{da} = c_1 \varepsilon R M \left(1 - u(a)\right) - \gamma M$$

$$\frac{dS}{da} = u(a) c_2 \varepsilon R M$$

Main assumptions

- spore production
- asexual reproduction
- no competition
- constant environment

State variables

R : Density of resources

M : Density of replication forms

S : Cumulative spore production

Parameters

$u(a)$: **Resource allocation strategy between within-host replication and spore production**

ε : Resource absorption efficiency

c_1 : Conversion rate of resources into replication forms

γ : Cellular mortality rate of replication forms

c_2 : Conversion rate of resources into spores

μ : External mortality rate (*e.g.* through host death)

Results

1- Optimal Resource Allocation Strategy is Bang-bang

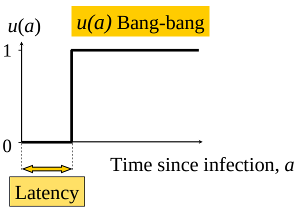

Fig 1: Diagram of the optimal resource allocation strategy

The optimal allocation strategy is first to allocate all consumed resources for replication only and after a critical date switch to the exclusive production of spores.

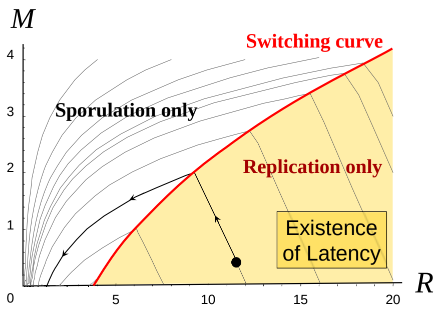

Fig 2: Trajectories followed by the pathogen in the phase plane defined by *R* and *M*

The duration of latency depends on initial conditions and parameter values, especially the balance between host exploitation efficiency (ε, c_1) and mortality factors (γ, μ).

2- Co-variation of Within-host Replication and Spore Production

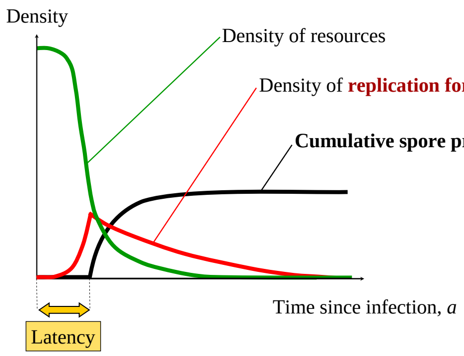

Fig 3: Dynamics of the state variables when the pathogen follow an optimal resource allocation strategy

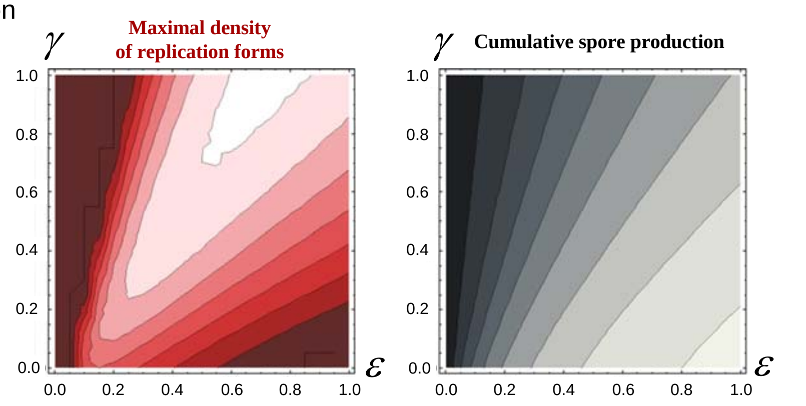

Fig 4: Maximal density of replication forms and Spore production as a function of resources absorption efficiency (ε) and cellular mortality rate (γ). **Lighter shading represents higher trait values**. c_1=0.6, c_2=1 and μ=0.2.

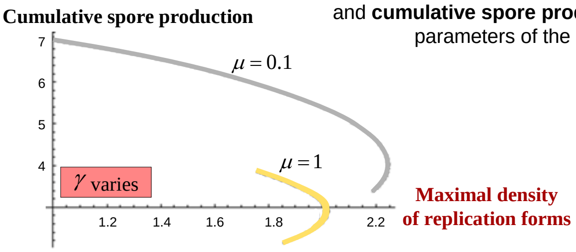

Fig 5: Covariation of within-host replication and spore production when cellular mortality rate (γ) varies

No clear correlation between **density of replication forms** and **cumulative spore production** is expected when the parameters of the model are let varied.

Conclusion

Although the trade-off between replication and multiplication does exist, it may be difficult to reveal it experimentally.

Assuming that the model parameters reflect the type of interaction between host and pathogen, their values will radically differ when confronting various host and parasite genotypes.

Therefore the strength and sign of the correlation between density in replication forms and spore production would depend on the individuals tested and the environmental conditions of the experiment.

Reference

[1] Gilchrist, M. A., Sulsky, D. L., Pringle, A. *Evolution* 2006.

INRA Nancy-Université Université Henri Poincaré Montpellier SupAgro CIRAD