Simulating realistic spatial structure for forest stands : a mimetic point process.

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Abstract

The recent development of individual based and spatially explicit models raises the question of how to simulate relevant initial states for these models, and more particularly how to simulate realistic spatial structures. This is especially true for forest stand modelling.

In this paper, we first review briefly the main simple steps in the simulation of forest stands with realistic spatial structure : (i) spatial structure analysis and ecological interpretation, (ii) expert simulation through classical point processes and *ad hoc* values of the parameters, and (iii) precise fitting of a spatial structure model using $L(r)$ and $L_{12}(r)$ functions as criterion. We illustrate the advantages and limits of these steps on a real forest stand in mixed Oak - Scots pine forest near Orléans (France).

We then propose a specific method - a mimetic point process - in order to simulate spatial structures similar to real patterns. This point process is based on a generalised Gibbs process, whose global cost function directly depends on the difference between the real measured values of $L(r)$ and $L_{12}(r)$ functions, and the corresponding values computed on the simulated pattern at each iteration of the algorithm. We applied this method on a mixed Oak - Scots pine forest stand, and thus obtained an improved simulated pattern, that can be used as initial state for individual based forest growth models.

We finally discuss the limits of this method, and its possible use to facilitate the valorisation of individual based models in the community of forest management.

Keywords : spatial pattern; simulation; point process; Gibbs process; individual based model; mixed forest; Oak; Scots pine

1. Introduction

The analysis of spatial structure is now commonly used in plant ecology (e.g. Tomppo 1986; Haase 1995; Pélissier & Goreaud 2001; Wiegand & Moloney 2004), where it allows to characterise the structure of different populations, as well as the spatial interactions between populations. It can also be used to infer some information on the biological processes, that are highly linked to the spatial structure of an ecosystem (Moeur 1993; Barot et al. 1999; Goreaud 2000). Indeed, the observed spatial structure results from past biological processes (especially birth and mortality of plants), and in return it defines the variety of local neighbourhoods of each plant, which will influence future processes such as competition and mortality.

In parallel with this interest for spatial structure, there has been recently a wide development of individual based models (Huston et al. 1988; Judson 1994; Grimm 1999), most of them being spatially explicit. In plant ecology, such models allow to consider explicitly the individual evolution of each plant, and to take various sources of local competition into account (Pacala & Silander 1985; Wu et al. 1985; Pukkala 1989; Grover 1997; Biondini 2001). Spatially explicit individual based growth models usually represent this local competition through various competition indexes, mostly based on the number, size, and location of neighbours of different species. Many models of that type have been developed recently in forest ecology (e.g. Ek & Monserud 1979; Tomé & Burkhart 1989; Houllier et al. 1991; Biging & Dobbertin 1995; Courbaud et al. 2001; Pretzsch et al. 2002).

In order to use such spatially explicit individual based models, it is necessary to define very precisely the initial state of the simulation at the individual level, with the location and characteristics of all plants. Indeed, the model will need this information to compute the competition indexes of each plants, and thus its evolution. For instance, if we want to simulate the evolution of a real forest stand, we will need a complete detailed map of the area, with the location and characteristics of all the trees.

However, such data are very expensive. Precise measurement of location of plants requires a huge amount of work, an is not always possible, especially when considering a large area. It is totally irrelevant in the context of forest management.

Therefore, forest modellers have developed some tools to simulate such individually detailed initial states, and allow the use of these models when no data are available (Tomppo 1986; Pretzsch 1997; Batista & Maguire 1998; Goreaud 2000). First models used - sometimes implicitly - very simple initial state simulations, based for instance on gaussian distribution of size and random location of plants (e.g. Pacala & Silander 1985; Pacala et al. 1993). More recently, modellers have tried to refine this initial state simulation, taking into account some well known characteristics of the considered plant communities. For instance, in forest growth models, foresters can sometimes choose the spatial structure of initial state between random, clumped or regular patterns, either to simulate a more realistic structure (Pretzsch 1997; de Coligny et al. 2004), or to test the influence of the initial structure on the model (e.g. Ménard et al. 2002; Couteron et al. 2003; Woodall & Graham 2004).

However, only few studies propose to really fit a model of spatial structure on real data, or to estimate the realism of the spatial structure simulated as initial states for forest growth models (Rathbun & Cressie 1994; Batista & Maguire 1998). This point is very important, because many studies have shown that the dynamics of a plant community can be very dependant on its spatial structure, especially in case of mixed species (e.g. Begon et al. 1990; Dieckmann et

al. 2000; Goreaud et al., 2002). Thus, using unrealistic structure as initial state of a simulation with a spatially explicit individual based model can often bring to unrealistic simulation results. The impact of the initial state on the dynamics of a model is in fact a general problem, and some authors simply decide not to consider the first part of a simulation, supposing it can be unrealistic. They then make the implicit assumption that the dynamics of the model will lead to a more realistic state after a short time period. However, this hypothesis is seldom tested, at least as far as spatial structure is concerned. Moreover, it is probably more relevant in natural forests that have reached an equilibrium than in managed stands. Therefore, we believe it is important to be able to simulate directly a realistic spatial structure for such virtual stands used as initial states for simulations with spatially explicit models.

Aim of the paper :

In this paper, we propose a specific method - a mimetic point process - to simulate realistic spatial structures similar to real patterns. We apply it to a mixed Oak - Scots pine forest near Orléans, which is presented in section 2. We first review very briefly the main simple methods to simulate realistic spatial patterns (section 3), and then detail the mimetic point process we propose (section 4). We finally discuss the limits of this method, and its possible use to facilitate the valorisation of individual based models in the community of forest management.

2. Material : Oak - Scots pine mixed stands

In order to illustrate the different methods presented in this paper, we used data from a mixed Oak - Scots pine forest near Orléans (France).

The Orléans state forest is one of the largest public woodlands in France (350 km2). It is located a few kilometres north of the Loire river. It is a typical flat, lowland forest, stretching from Gien to Orléans. Our experimental network was set up in the southern part of the forest, characterised by alluvial deposits of sand upon clay. The absence of natural drainage and the non permeable substratum both contribute to the development of groundwater tables. Soils are often acidic and characterised by successive water-logging and dry periods.
The ancient Oak forest was heavily over-harvested from the Middle Ages to 1850. By that time, the Oak stands had gone to pieces, gaps were very numerous and accounted for 30% of the whole surface. Between 1870 and 1890, the oppenings had been reforested by either seeding or planting Scots pine. In this way, the administration intended to reclaim soils and to restore the forest ecosystem. Scots pine found good growing conditions, and furthermore Oak was regaining its ecological and economic interest. Since that time, Oak and Pine have been managed together. Nowadays, foresters want to continue to manage the large area that originated from the second generation of Pine as mixed stands. These stands are the object of our surveys.

We have set up and mapped twenty 1ha plots in order to study the spatial structure of these mixed Oak - Scots pine forest stands (Allain et al. 2004; Ngo Bieng 2004). In each plot, we have measured the precise location of each tree (diameter>10cm), as well as its species, circumference, and storey. For this paper, we chose to use plot n°12 (figure 1), which corresponds to a simple case of mixed stand, and is representative of one type of the mixed stands in this forest (Ngo Bieng 2004).
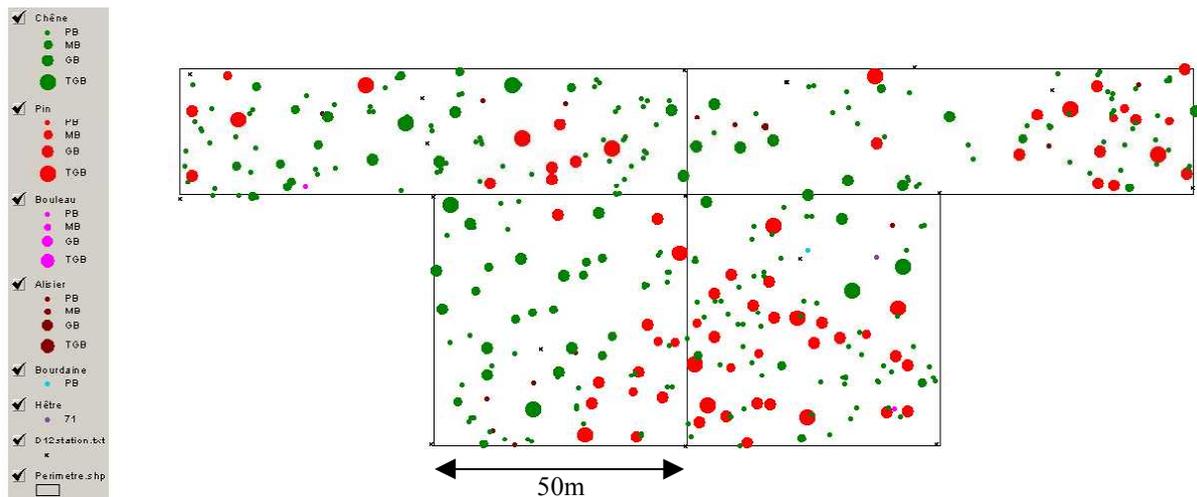
Figure 1 : Location of trees in plot 12 in Orléans forest. The different species are represented by different colours (oaks in green, Scots pines in red), and the size of the dots corresponds to different diameter classes.

3. From spatial structure analysis to forest stand simulation.

Many methods have been used to simulate a more or less realistic spatial structure for forest stands, or more generally plant communities (Tomppo 1986; Rathbun & Cressie 1994; Pretzsch 1997; Batista & Maguire 1998; Goreaud 2000; Ménard et al. 2002; Couteron et al. 2003; Woodall & Graham 2004). We present here what seems to us to be the 3 main classical steps in such spatial structure simulation. We illustrate each of these steps with the plot n°12 in the Orléans forest, already presented in section 2.

31. Spatial structure analysis.

If we want to simulate the spatial structure of a forest stand realistically, the first step is of course to analyse this structure as precisely as possible (Batista & Maguire 1998). The analysis of spatial patterns has been more and more commonly used in ecological literature. Different methods can be used, depending on the data available (e.g. Ripley 1981, Diggle 1983, Cressie 1993). The best situation corresponds to cases where at least one completely mapped plot is available in the considered forest stand, which will allow to analyse the spatial structure in detail. In other cases, some measures of nearest neighbours distances can be used.

The results of this analysis will help us to define the main characteristics of the spatial structure we will then have to simulate (especially : aggregation, regularity or randomness). As trees of different species or ages can a priori have different spatial structure, it can be relevant to define some homogeneous types of trees, and then to analyse the specific structure of each type, as well as the spatial interactions between types. This is especially true if the stand is mixed (different species), or irregular (trees of different age or size) : we can define a type for each species, or even for each size - or age - class in each species. In this paper, we will call "sub-population" the set of trees corresponding to a given type.

Let us now consider the plot 12 in the Orléans forest. In this plot, we defined three sub-populations. First, as all Scots pines are in the canopy, they can be considered as one sub-population as a whole. Second, we have analysed precisely the spatial structure of oaks (Ngo Bieng 2004) and have shown that it was possible to classify them in only two sub-populations, corresponding to oaks from the canopy, and oaks from the understorey, respectively. We considered that oaks from the canopy also correspond to older trees. There are very few trees of other species in this plot, so we decided to neglect them.

As we had a complete map of this plot (figure 1), we used the classical $L(r)$ function (Besag 1977, Ripley 1977) and $L_{12}(r)$ intertype function (Lotwick & Silverman 1982) to characterise the specific spatial structure of each sub-population, and the structure of the interactions between sub-populations, respectively. Indeed, these functions characterise the spatial structure of point patterns at different scales, and are more and more commonly used in ecological studies. We used the classical null hypothesis of Complete Spatial Randomness (CSR) for $L(r)$, and the null hypothesis of Population Independence for $L_{12}(r),$ because our three sub-populations correspond to *a priori* different patterns. For each range r, we computed the corresponding confidence intervals with a risk $\alpha=1\%$, using Monte Carlo simulations. More details on the methods and programs used to estimate these functions and their confidence intervals can be found in (Goreaud & Pélissier 1999; Goreaud & Pélissier 2003).

Figure 2 presents the results of the analyses we made on these three sub-populations.
The spatial structure of the canopy is the direct result of the sylvicultural history. Scots pines and oaks both have clumped spatial structures (significant positive values of the $L(r)$ function at different ranges r), and present an interspecific repulsion (significant negative values of the $L_{12}(r)$ function up to $r=10$m), which corresponds to the fact that Scots pines have been planted or seeded in gaps where there were no oaks. We can note that the $L(r)$ function for Scots pines has a very classical shape, with a clear maximum around 20m, whereas the $L(r)$ function for oaks from the canopy remains positive over $r=50$m, which could correspond to some heterogeneity of the pattern.
As far as oaks from the understorey are concerned, they also have a clumped structure, with a maximum around 14m. The high values of $L(r)$ for $r=2$m corresponds to double-stemmed trees. The intertype functions show a tendency toward repulsion with oaks from the canopy, and attraction with Scots pines, which may correspond to the fact that there is more light available under Scots pines for regeneration and growth.
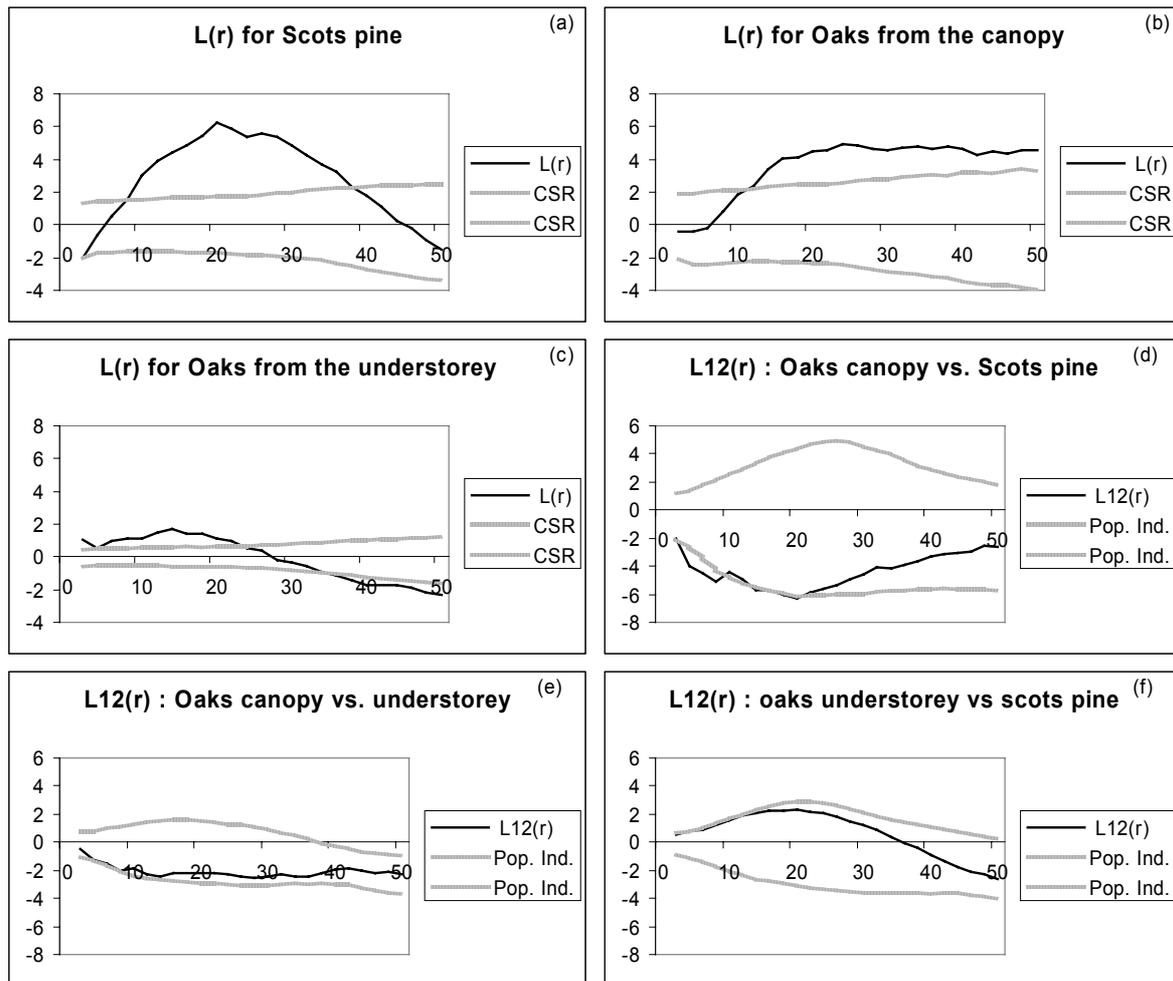
Figure 2 : Analysis of the spatial structure of plot 12 in Orléans forest : *L(r)* function (black line), and the corresponding confidence interval for the C.S.R. null hypothesis (grey line), for the three sub-populations : (a) Scots pines, (b) oaks from the canopy, (c) oaks from the understorey. Intertype $L_{12}(r)$ function (black line), and the corresponding confidence interval for the Population Independence null hypothesis (grey line), for (d) oaks vs. Scots pines in the canopy, (e) oaks from the canopy vs. the understorey, and (f) oaks from the understorey vs. Scots pines.

32. Expert simulation

Once the spatial structure of the stand is analysed, we can use its main characteristics in order to chose relevant point processes to simulate similar structures. Very classically, Poisson process will be used to simulate patterns whose structure is not significantly different from Complete Spatial Randomness (CSR); Neyman-Scott processes can be used to simulate easily clumped patterns; and Gibbs processes to simulate regular or more complex patterns (Diggle 1983; Tomppo 1986). Some models use such point processes to offer more possibilities to users when simulating the initial state of a forest dynamics simulators. This is especially the case with the structure generator of Selva model : STRUGEN (Pretsch 1997) that can simulate very complex spatial patterns for managed stands. The CAPSIS plateform (de Coligny et al. 2004) also proposes some tools to simulate virtual stands of different spatial patterns, especially with the Mountain model (Courbaud et al. 2001).

Of course, when the population is composed of different sub-populations, each sub-population can be simulated by a specific point process. Intertype relations can be taken into account, for instance in generalised Gibbs processes. In this case, the hierarchy of the interactions can have an influence on the order in which the sub populations have to be simulated (Högmander & Särkkä 1999). Ecological considerations can then be used, for instance to determine which sub-population is older, and thus has not been influenced by the location of younger ones.

Let us now consider the plot n°12 in the Orléans forest. The Scots pines sub-population presents a very clearly clumped pattern, and therefore we tried to simulate it with a very simple Neyman-Scott point process. We simulated the location of the N_{ag} centres of aggregates with a Poisson process. Then, for each aggregate, we simulated the location of exactly N_{tree} trees with another Poisson process within a disc of radius R. The overall study area is considered as a torus. We used the results of the spatial structure analysis (cf. 31) to define relevant values for the parameters. First, for a Neyman-Scott process, the range r corresponding to the maximum value of $L(r)$ is supposed to be slightly greater than the radius of the aggregates. As this maximum value for Scots pines occurs for about 20m, we chose R=15m. This corresponds to aggregates of about 707m² area. We then used the map (figure 1) to estimate the density inside an aggregate. The 25m*25m top right square of the plot contains 12 Scots pines, which corresponds to a density of 192 trees/ha, and thus to 13.5 trees per aggregate. As we want to simulate a total population of 69 trees per ha, we finally simulated the Scots pines sub-population with a Neyman-Scott process of N_{ag}=5 aggregates of N_{tree}=14 trees.

Figure 3 illustrates one realisation of this point process, on a 1ha square plot, and the corresponding $L(r)$ function, compared to the real values obtained on plot 12. The $L(r)$ function is not exactly the same, but there is a general similitude in shape.
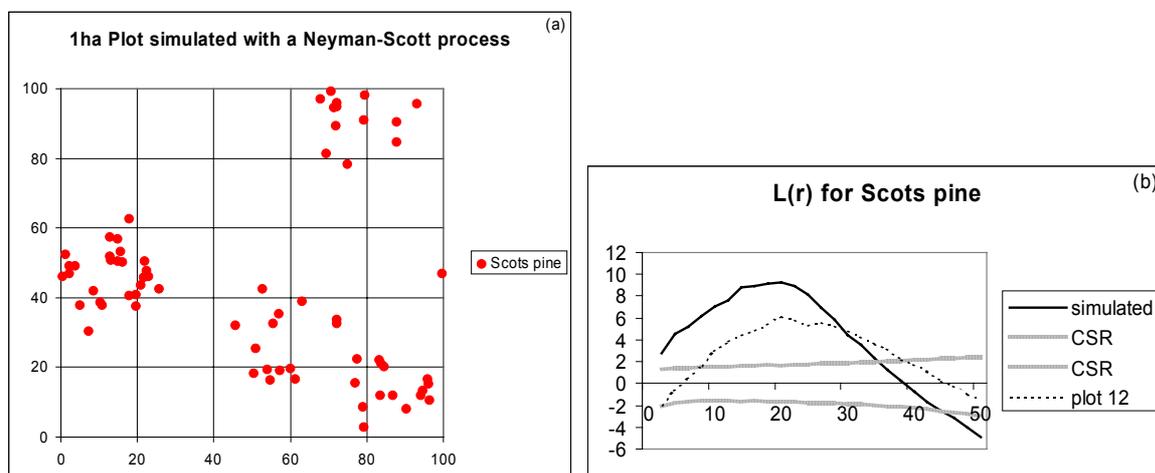

Figure 3 : (a) One realisation of a simple Neyman-Scott process simulating 70 Scots pine on 1ha ; and (b) the corresponding $L(r)$ function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of $L(r)$ for Scots pines measured on plot 12 (dotted line).

Similarly, taking into account the interspecific repulsion between oaks and Scots pines in the canopy, we simulated the 51 oaks from the canopy with a repulsive Gibbs process. As the intertype function (figure 2d) has very significant negative values for $r \leq 10m$, we simply defined our Gibbs process with a positive pairwise interaction function for $r \leq 10m$ (equation 1). We used 400 iterations of the classical stepwise depletion-replacement algorithm to

simulate this Gibbs process (Ripley, 1979). With this algorithm, the intensity of the repulsion increases with the number of iterations, and Ripley (1979) suggests to use a number of iterations equals to four times the number of points.

Figure 4 illustrates one realisation of this second point process, on the same 1ha square plot, and the corresponding *L(r)* and *L₁₂(r)* functions. As far as the intertype function is concerned, the results are more negative than in the real plot, but the shape is quite similar, and the relative position to the confidence interval too. The *L(r)* function for this sub population presents significant positive values, corresponding to the clumping effect of the repulsion with Scots pines, but the range of aggregation slightly differs with the range observed in plot 12.

Equation 1 : $f(r) = \begin{cases} 100 & if \ r \le 10m \\ 0 & otherwise \end{cases}$
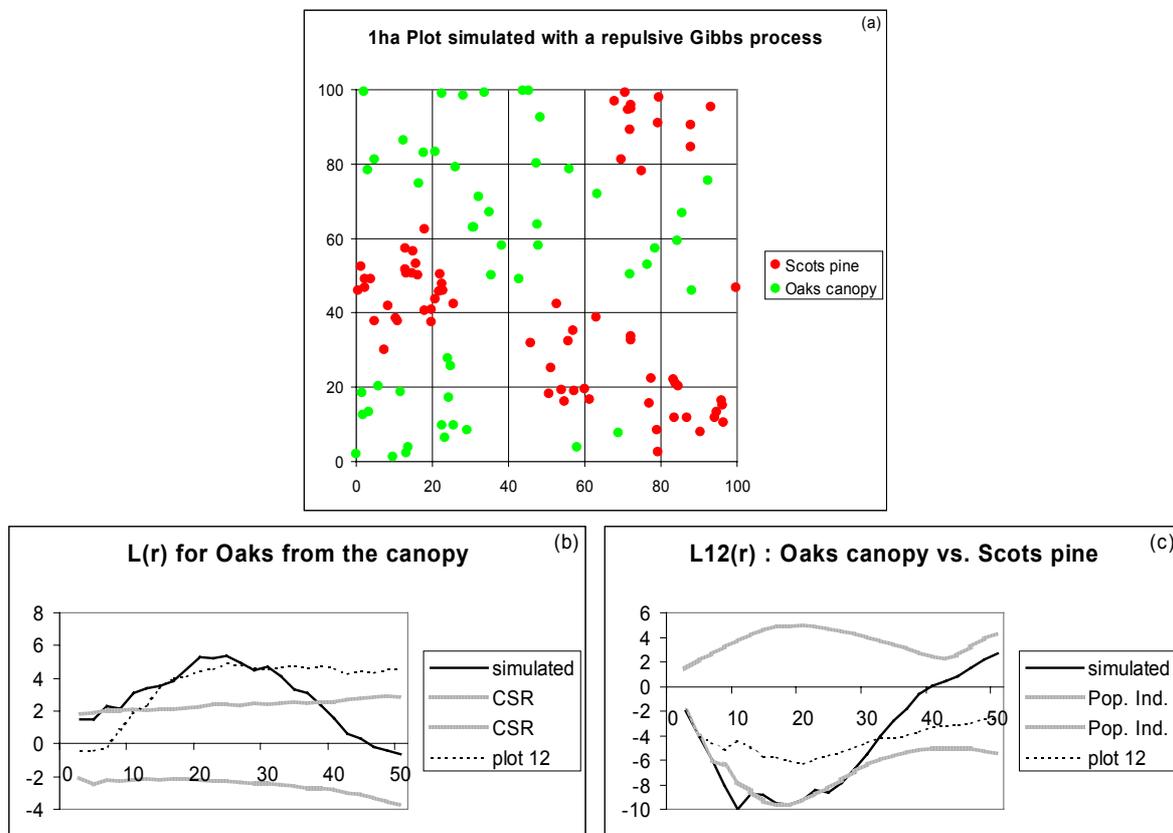

Figure 4 : (a) One realisation of a repulsive Gibbs process simulating oaks from the canopy in repulsion with Scots pines in a 1ha plot; (b) the corresponding *L(r)* function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of *L(r)* for oaks from the canopy measured on plot 12 (dotted line); and (c) the intertype function corresponding to the interaction between oaks in the canopy and Scots pines (bold black line) with the confidence interval for the null hypothesis of Population Independence (bold grey), compared to the real values of *L₁₂(r)* measured on plot 12 (dotted line).

The last sub-population, oaks from the understorey, is more difficult to simulate. As the intertype functions only show non significant tendencies (figure 2e and 2f), we could simply have simulated the clumped pattern (figure 2c) without any interaction with other sub-populations. However, the tendencies of repulsion towards other oaks, and attraction towards Scots pines make sense in an ecological point of view, because of the competition for light. Therefore, as we consider here a simulation through expertise, we chose to take them into account.

We used another Gibbs process to simulate the 212 oaks from the understorey. We defined an attraction towards Scots pines using a negative pairwise interaction function (equation 2). As we wanted the final pattern to be only very slightly structured, we used only 200 iterations of the algorithm, i.e. much less than the number of iterations used to simulate the oaks from the canopy.

Figure 5 illustrates one realisation of this third point process, and the corresponding $L(r)$ and $L_{12}(r)$ functions. As far as the intertype functions are concerned, the results are a little more significant, but not too different from the real plot, at least when considering the relative position to the confidence interval. The $L(r)$ function presents far more positive values than in plot 12, but the general shape is similar.

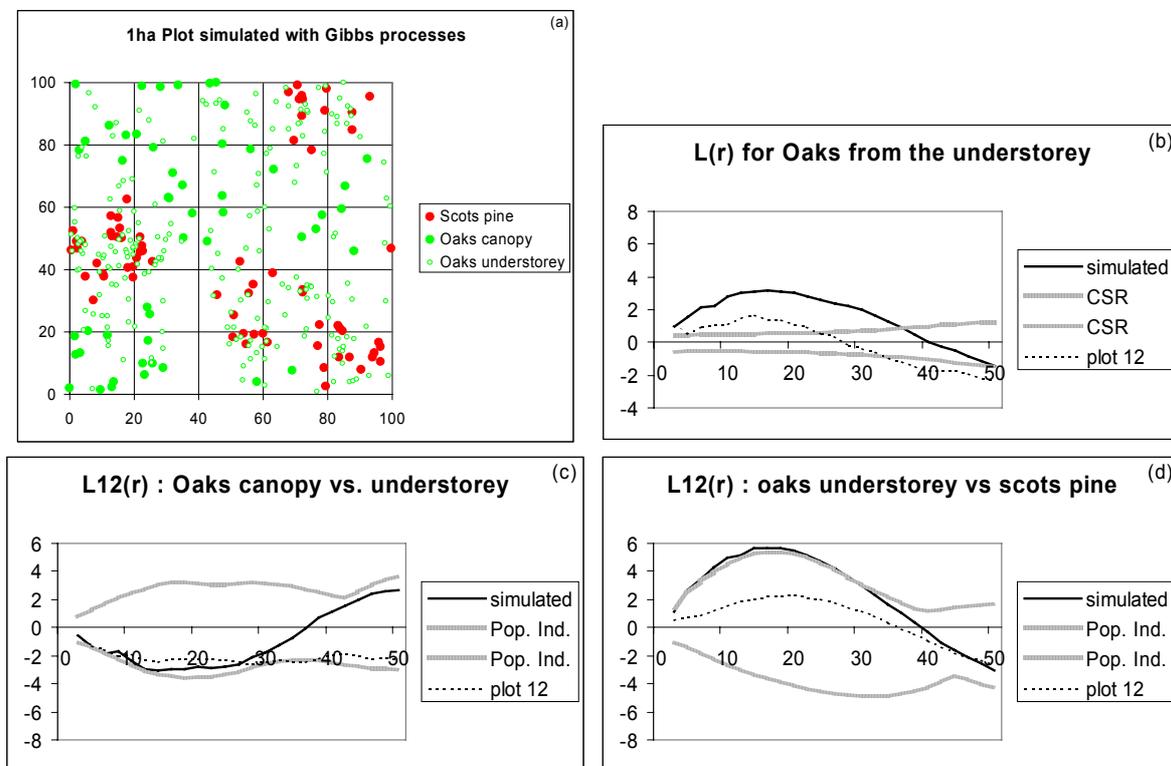Equation 2 : $$f(r) = \begin{cases} -100 & if\ r \le 10m \\ 0 & otherwise \end{cases}$$

Figure 5 : (a) One realisation of an attractive Gibbs process simulating oaks from the understorey in attraction with Scots pines in a 1ha plot; (b) the corresponding $L(r)$ function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of $L(r)$ measured on plot 12 (dotted line); and the intertype function (bold black line) corresponding to the interaction between oaks from the understorey and Scots pines (c), or between oaks from the canopy and oaks from the understorey (d), respectively, with their confidence interval for the null hypothesis of Population Independence (bold grey), compared to the real values of $L_{12}(r)$ measured on plot 12 (dotted line).

Finally, our expertise on the spatial structure in plot 12 allowed us to choose simple point processes, and *ad hoc* values of the parameters, in order to simulate forest stands of similar structure, taking both into account the specific spatial structure of each sub-population, and the intertype interactions between sub-populations. The result is not too bad, because the general shape of the $L(r)$ and $L_{12}(r)$ functions for the simulated patterns is similar to the corresponding functions computed on plot 12. However, we must also recognise that there are

some differences : the simulated spatial structure of Scots pines seems to be more intensely clumped than the real one ; we did not succeed in simulating the positive values of *L(r)* up to 50m for oaks from the canopy ; and the intertype attraction between simulated oaks from the understorey and Scots pines is too intense. We can therefore suspect that the *ad hoc* values of the parameters we used are not the best possible ones. In the next section, we will see how this point can be improved.

33. Fitting simple models

In the "expert simulation" step presented in the previous section, we used our expertise in spatial structure analysis, and our knowledge on the dynamics of the stand, to choose relevant point processes to simulate realistic spatial structures. We thus in fact built a model of this spatial structure. However, we also used only our expertise to define more or less realistic parameter values. Therefore, the result is somehow disappointing : the general shape of the curves are similar to our real stand, but the spatial structure is different in many details.

In order to improve this point, we can try to find better values of the parameters, which means in fact simply to fit our model of spatial structure. As we have real values of the *L(r)* and $L_{12}(r)$ functions, measured on our real plot, we can consider them as our real data, and fit our model so that the simulated *L(r)* and $L_{12}(r)$ functions are as close as possible to the real values (Diggle 1983). In the literature, only few papers propose to really fit some point patterns to real data. Rathbun & Cressie (1994) have fitted a space time point process to model both the spatial structure and the dynamics of a forest stand. Batista & Maguire (1998) have fitted a few models of spatial structure in tropical forest, using the modified least square criterion proposed by Diggle (1983). Goreaud (2000) has fitted a doubly aggregated Neyman-Scott point process on storm mortality data. Särkkä (1995), Särkkä & Tomppo (1998) have used the pseudo likelihood approach to fit Gibbs processes on forest plots.

Let us now consider plot 12 in the Orléans forest. In section 32, we have chosen a Neyman-Scott process to model the spatial structure of the Scots pines sub-population. This very simple point process only has 3 parameters : N_{ag}, the number of aggregates, *R*, the radius of aggregates, and N_{tree}, the number of trees in each aggregate. As we want to simulate a fixed total number of Scots pines (69), we can consider that N_{tree} can be deduced from N_{ag}, and thus we only have to estimate N_{ag} and *R*.

We wanted to estimate the values of the parameters that best correspond to the real spatial structure of Scots pines in plot 12. We fitted our Neyman-Scott point process so that the *L(r)* function of the simulated pattern ($L^{sim}(r)$) be as close as possible to the real values of *L(r)* for Scots pines ($L^{real}(r)$). We used the least square criterion defined in equation 3. Unfortunately, we could not calculate the theoretical value of $L^{sim}(r)$. Although some analytical results can be found for Neyman-Scott processes under some hypotheses (Diggle 1983; Goreaud 2000), they did not exactly correspond to the simulated process used in this paper (especially because of torus correction). We therefore had to use Monte Carlo simulations.

Equation 3 :   *Fitting criterion* $= \displaystyle\sum_{r=2}^{50} \left(L^{real}(r) - L^{sim}(r) \right)^2$

For each tested value of N_{ag} (from 1 to 20) and R (from 1 to 40), we simulated 1000 realisations of the corresponding Neyman-Scott process, and we estimated the fitting criterion for each realisation. We then compared the mean value of the criterion for each set of parameters. We obtained the lowest fitting criterion (FC=156) for N_{ag}=8 & R=21m (leading to N_{tree}=9). Figure 6 shows one realisation of the corresponding point process, on a 1ha square plot, with its $L(r)$ function, compared to the real values obtained on plot 12. The spatial structure of this realisation seems indeed to be very similar to the real spatial structure of Scots pines, and at least more similar than the spatial structure simulated through expertise in section 32 (figure 3).
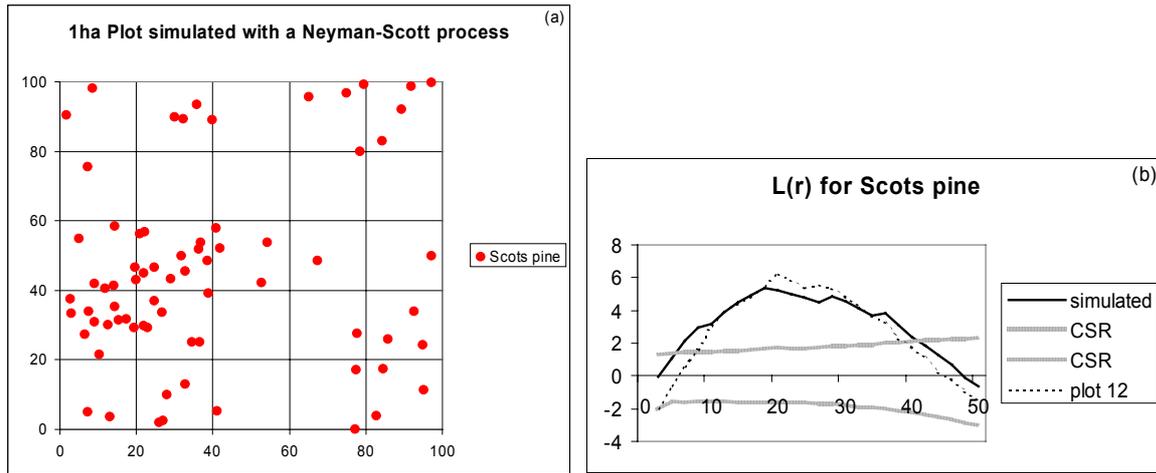

Figure 6 : (a) One realisation of the fitted Neyman-Scott process simulating 72 Scots pines on 1ha ; and (b) the corresponding $L(r)$ function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of $L(r)$ for Scots pines measured on plot 12 (dotted line).

We applied exactly the same method to fit the parameters of the repulsive Gibbs process used to simulate the 51 oaks from the canopy in repulsion with Scots pines. As we wanted both the specific spatial structure of oaks, but also the intertype structure to be as realistic as possible, we used a more complex fitting criterion (equation 4) including the difference between the real and simulated values for both $L(r)$ and $L_{12}(r)$ functions.

Equation 4 : $$Fitting\ criterion = \sum_{r=2}^{50}(L^{real}(r) - L^{sim}(r))^2 + \sum_{r=2}^{50}(L_{12}^{real}(r) - L_{12}^{sim}(r))^2$$

Gibbs processes are complex processes, whose parameters include the pairwise interaction function. Precise fitting of these parameters require to define the possible values of the pairwise interaction function for different ranges, and to fit each of them (Särkkä 1995; Goulard et al. 1996). In this paper, we did not go so far, and we only considered very simplified repulsive Gibbs processes with a constant positive value (100) for a given range R (equation 5). We thus only had to estimate 2 parameters for the simulated process : the range R of this interaction, and the number of iteration N_{iter} of the depletion replacement algorithm, which is linked to the intensity of the simulated repulsion.

Equation 5 : $f(r) = \begin{cases} 100 & if\ r \leq R \\ 0 & otherwise \end{cases}$

For each tested value of N_{iter} (from 50 to 500) and R (from 1 to 20), we simulated 1000 realisations of the corresponding Gibbs process. Scots pine were represented by the realisation of the previously fitted Neyman-Scott process shown in figure 6. We estimated and compared the mean value of the fitting criterion for each set of parameters. We obtained the lowest fitting criterion (FC=179) for N_{iter}=100 & R=10m. Figure 7 illustrates one realisation of the corresponding repulsive Gibbs process, with its $L(r)$ function, compared to the real values obtained on plot 12, and the intertype $L_{12}(r)$ function corresponding to the spatial interactions between oaks and Scots pines in the canopy. In this realisation, the spatial interaction between oaks and Scots pines in the canopy is indeed very similar to the real interaction observed in plot 12, and at least better than what we obtained through expertise in section 32 (figure 4). However, this new fitted Gibbs process does not simulate very precisely the specific spatial structure of oaks from the canopy, which seems even more different from plot 12 than in the previous section. This probably results from the fact that the repulsive Gibbs process we used only take the intertype structure into account, and is therefore not relevant to simulate the specific spatial structure of oaks in the canopy, even when this specific spatial structure is included in the fitting criterion.
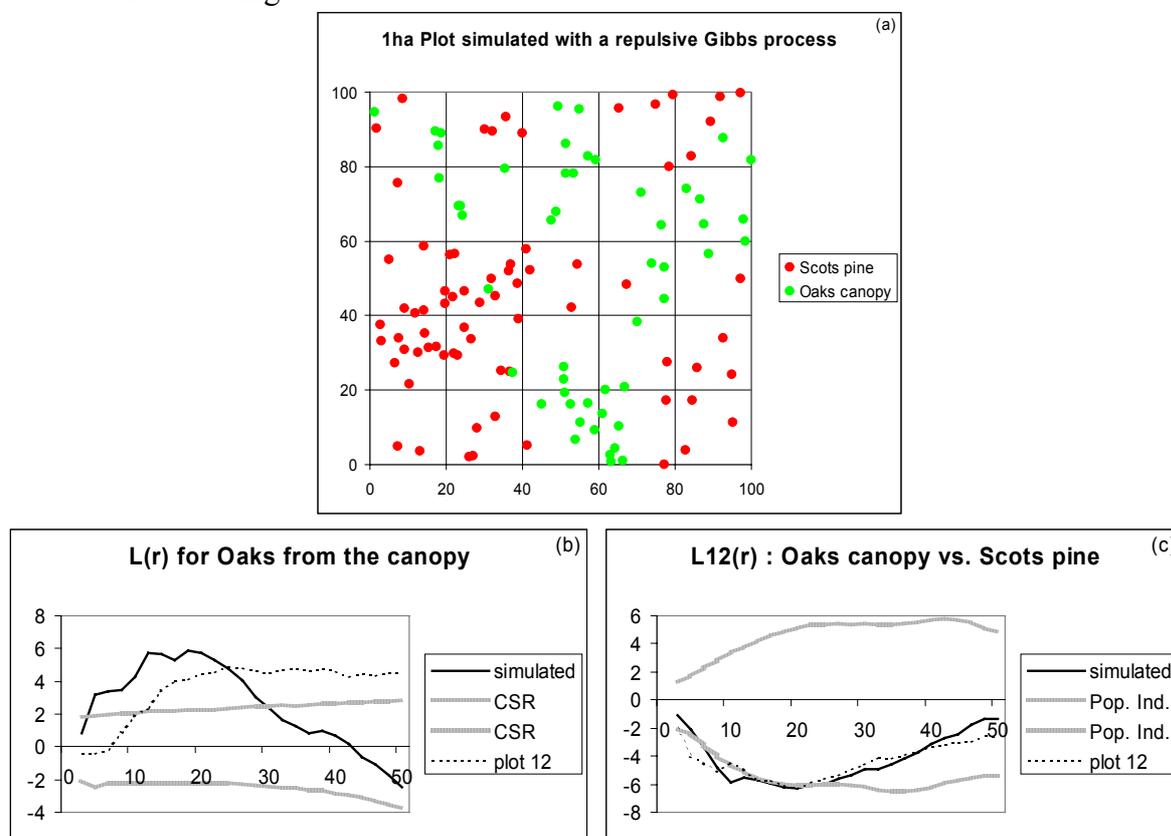

Figure 7 : (a) One realisation of the fitted repulsive Gibbs process simulating oaks from the canopy in repulsion with Scots pines in a 1ha plot; (b) the corresponding $L(r)$ function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of $L(r)$ for oaks from the canopy measured on plot 12 (dotted line); and (c) the intertype function corresponding to the interaction between oaks in the canopy and Scots pines (bold black line) with the confidence interval for the null hypothesis of Population Independence (bold grey), compared to the real values of $L_{12}(r)$ measured on plot 12 (dotted line).

We applied the same method again to fit the parameters of the attractive Gibbs process used to simulate the 212 oaks from the understorey in attraction with Scots pines. The fitting criterion (equation 6) now includes the $L(r)$ function for this third sub-population, but also the two intertype functions corresponding to its interactions with both Scots pines ($L_{12}(r)$) and oaks from the canopy ($L_{13}(r)$).

equation 6 : $\sum_{r=2}^{50}(L^{real}(r)-L^{sim}(r))^2 + \sum_{r=2}^{50}(L_{12}^{real}(r)-L_{12}^{sim}(r))^2 + \sum_{r=2}^{50}(L_{13}^{real}(r)-L_{13}^{sim}(r))^2$

For each value of N_{iter} (from 50 to 500) and R (from 1 to 20), we simulated 1000 realisations of the corresponding Gibbs process. Scots pines and oaks from the canopy were represented by the realisations of the fitted processes shown in figure 7. We obtained the lowest mean fitting criterion (FC=112) for N_{iter}=300 & R=2m. Figure 8 illustrates one realisation of the corresponding repulsive Gibbs process, with its $L(r)$ function, compared to the real values obtained on plot 12, and the intertype $L_{12}(r)$ functions corresponding to its spatial interactions with oaks from the canopy and Scots pines, respectively. The result of this simulation can seem disappointing, because none of the three functions looks really similar to the real values, not even in shape or compared to the confidence interval. However, the value of the fitting criterion obtained for this realisation (FC=140) is not so bad compared to mean values obtained for the two other sub-populations (FC=156 and 179, see above). Moreover, it is far better than the corresponding value computed on the realisation obtained through expertise for oaks from the understorey (figure 5, FC=322). The problem is that our fitting criterion only takes into account the differences between the real and simulated values of the functions, not their shape or their relative position to the confidence interval. We can suppose that the attractive Gibbs process we used was in fact not relevant to simulate the complex spatial structure of this sub-population. Even if the fitted point process minimises the differences, it can not simulate the patterns properly
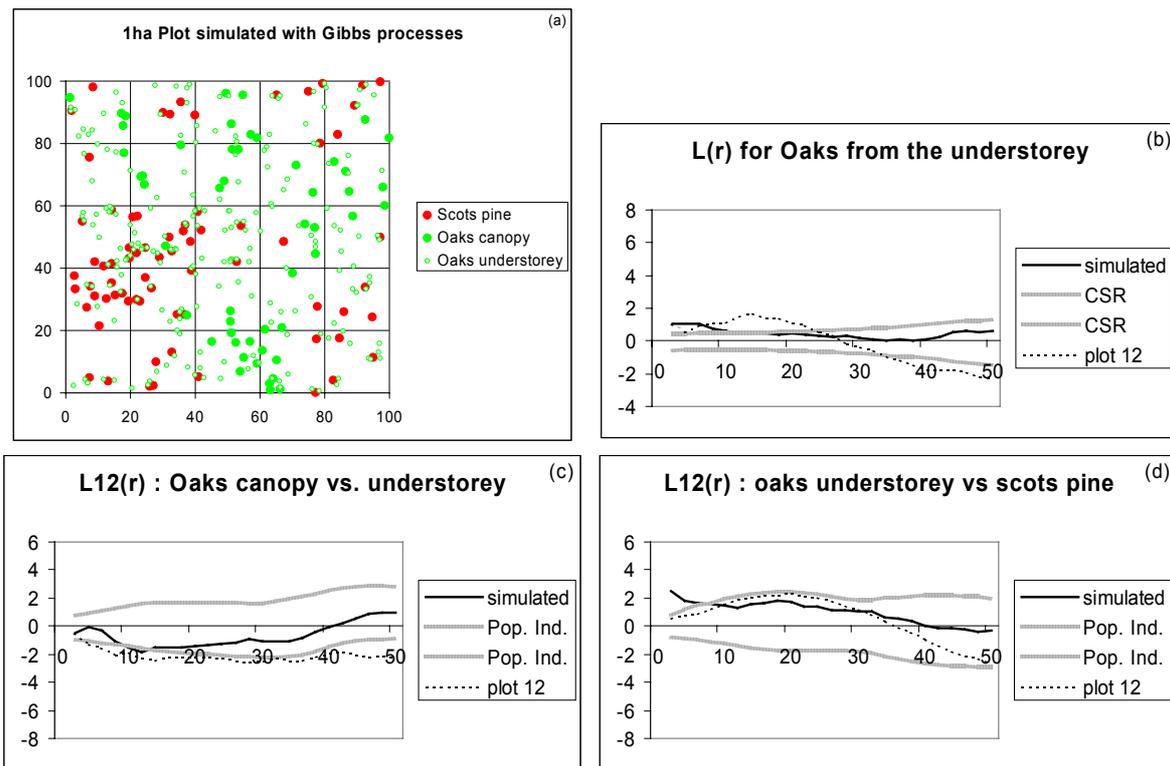

Figure 8 : (a) One realisation of the fitted attractive Gibbs process simulating oaks from the understorey in attraction with Scots pines in a 1ha plot; (b) the corresponding $L(r)$ function (bold black line) with its CSR confidence interval (bold grey), compared to the real values of $L(r)$ for oaks from the understorey measured on plot 12 (dotted line); and the intertype function (bold black line) corresponding to the interaction between oaks from the understorey and Scots pines (c), or between oaks from the canopy and oaks from the understorey (d), respectively, with their confidence interval for the null hypothesis of Population Independence (bold grey), compared to the real values of $L_{12}(r)$ measured on plot 12 (dotted line).

Finally, fitting the point processes with an *ad hoc* fitting criterion, based on the differences between the real and simulated values of the $L(r)$ and $L_{12}(r)$ functions allowed us to improve significantly the realism of our simulated patterns, at least in terms of the similarity of these functions that describe the main characteristics of the spatial structure of the stand.

However, the fitting method we used is not completely satisfactory. Indeed, it requires Monte Carlo simulations of the considered point process in order to estimate the fitting criterion for each values of the parameters. Therefore, there is some stochasticity in the fitting method itself, and the optimal values of the parameters obtained can vary from one run to another. We tried to reduce this variability by computing the mean value of the fitting criterion for 1000 Monte Carlo simulations of the point process, but this seems not to be really sufficient. Indeed, when repeating the same fitting procedure for the three point processes used in this paper, we obtained small differences in the optimal parameters. A more precise analysis of this variability would be required in order to define the proper number of Monte Carlo simulations to be used.

Our results also points out the fact that we are always limited by the choice of the point process we use to model a given spatial pattern. Even when optimising the values of the parameters, a given point process can not simulate any spatial structure. The repulsive Gibbs process we used to simulate oaks from the canopy was able to simulate the intertype function correctly, but not the specific spatial structure observed. The attractive Gibbs process we used to simulate oaks from the understorey was not able to simulate the general shape of the $L(r)$, nor of the $L_{12}(r)$ functions. Batista & Maguire (1998) had similar conclusions on their fitted point processes in tropical forest.

In the next section, we propose another approach, based on a mimetic point process, that partially solves these problems and improves the realism of the simulated pattern.

4. A mimetic point process to simulate realistic structures

In this section, we propose another method to simulate realistic complex patterns similar to real patterns measured in forest stands. We first detail the general principles of this method (section 41), a mimetic point process obtained by using the difference between the real and simulated values of $L(r)$ and $L_{12}(r)$ functions as global cost in a generalised Gibbs process. We then illustrate this method on plot n°12 in the Orléans forest (section 42).

41. General principle

We have seen in section 3 that it is possible to use the characteristics of the spatial structure of a forest stand - for instance the $L(r)$ and $L_{12}(r)$ functions - in order to improve the realism of a simulated pattern. In particular, we have fitted our structure model using an *ad hoc* criterion (equation 3, 4, 6), based on the difference between the real measured values of $L(r)$ and $L_{12}(r)$ functions and the corresponding values computed on the simulated pattern.

The main idea of the mimetic point process we propose here is to use this criterion not only as an external fitting criterion, but directly inside the global cost function of a generalised Gibbs

process. We therefore intend to simulate a point pattern whose $L(r)$ and $L_{12}(r)$ functions are as close as possible to the real measured ones, which we will consider as a proof that the simulated structure is realistic. Note that this method is very general, and could be adapted to other characteristics of the spatial structure, for instance $K(r)$ and $K_{12}(r)$ functions, distances to nearest neighbours, or any usual index used to analyse the spatial structure. It would only require to modify the global cost function inside the mimetic process.

In a classical Gibbs process simulation, a global cost function is defined as the sum of local costs for all couples of points (Tomppo 1986). These local costs are estimated by a pairwise interaction function, whose values are the parameters of the process. Depending on their sign, the minimisation of the global cost leads either to repulsion or to attraction between points.

As we intended to obtain a pattern whose $L(r)$ and $L_{12}(r)$ functions are as similar as possible to the corresponding characteristics measured on a real plot, we decided to define the global cost in our generalised Gibbs process directly as a measure of the difference between these real and simulated values (equation 7). Therefore, when minimising the global cost, for instance with the usual stepwise depletion-replacement algorithm, we will obtain a simulated pattern whose characteristics become more and more similar to the real one.

equation 7 : $\quad GlobalCost = \sum_i \left\| L_i^{real}(r) - L_i^{simulated}(r) \right\|^2 + \sum_{i>j} \left\| L_{i.j}^{real}(r) - L_{i.j}^{simulated}(r) \right\|^2$

Note that such a global cost is not expressed as a sum of pairwise interaction functions, so that our mimetic process is not exactly a Gibbs process in the usual meaning of the term.

42. Application on plot 12.

We have implemented this algorithm (in C++), using $L(r)$ and $L_{12}(r)$ functions as characteristics of spatial structure, and applied it to plot 12. We have considered the three sub-populations defined in section 3, and simulated another 1ha plot with the same number of trees : 69 Scots pines, 51 oaks from the canopy, and 212 oaks from the understorey. For each sub-population, and each intertype interaction, we have computed 25 values of $L(r)$ and $L_{12}(r)$ functions, respectively, corresponding to radius 2,4,..., 50m.

We first simulated a Poisson initial state, with independence between the sub-populations. We then iterated the depletion-replacement algorithm, drawing one tree at random, and comparing its actual location to 4 other random locations. For each of these locations, we computed the new global cost following equation 8. We then kept the location corresponding to the smaller cost.

Equation 8 : $\quad GlobalCost = \sum_{i=A,B,C} \left(\sum_{r=2}^{50} \left(L_i^{real}(r) - L_i^{sim}(r) \right)^2 \right) + \sum_{\substack{i,j \in \{A,B,C\} \\ i<j}} \left(\sum_{r=2}^{50} \left(L_{i.j}^{real}(r) - L_{i.j}^{sim}(r) \right)^2 \right)$

This algorithm leads to a fast decrease of the global cost : for instance with an initial cost of 1455, we obtained a cost of 60 after 1000 iteration, and the global cost then stabilised progressively (figure 9). We stopped the algorithm after 5000 iterations (cost = 18). Figure 10

shows one realisation of this mimetic process, and figure 11 the corresponding $L(r)$ and $L_{12}(r)$ functions compared to the real values measured on plot 12.
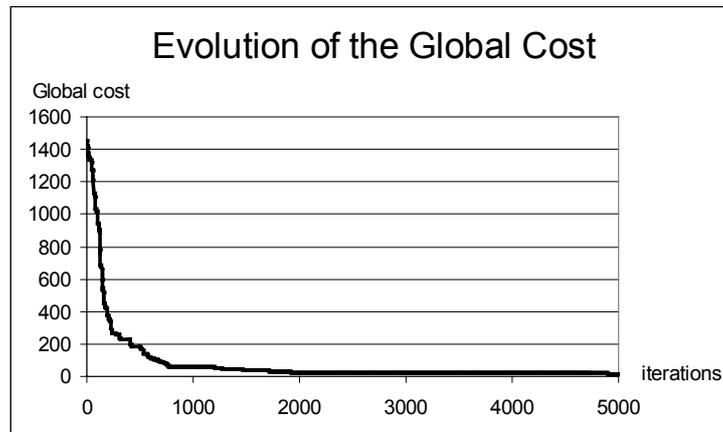

Figure 9 : Evolution of the global cost of the mimetic process (equation 8) through the iterations of the depletion replacement algorithm.
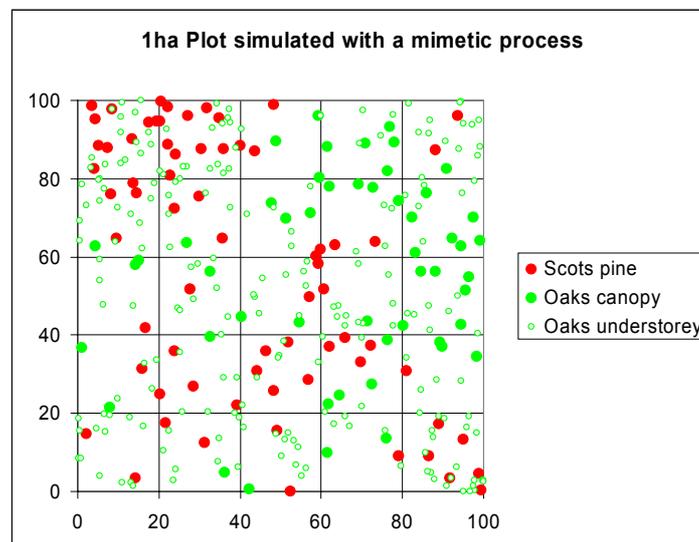

Figure 10 : One realisation of the mimetic process on 1ha.

The simulated values are indeed really close to the real ones, which corresponds to the very small value of the global cost. This confirms the ability of the mimetic process to simulate realistic patterns, taking both the spatial structure of each sub-population and the spatial interactions between sub-populations into account. If we consider the $L(r)$ and $L_{12}(r)$ functions as good criteria to characterise the spatial structure of a forest stand, then this method is an improvement compared to the classical methods illustrated in section 3. Indeed, the global cost of the mimetic process corresponds to the sum of the three fitting criterion used in section 33, which mean value is 447, and was thus far higher for the realisations obtained through process fitting than for the realisations of the mimetic process.

However, this method rises many methodological questions, that we discuss in section 5.
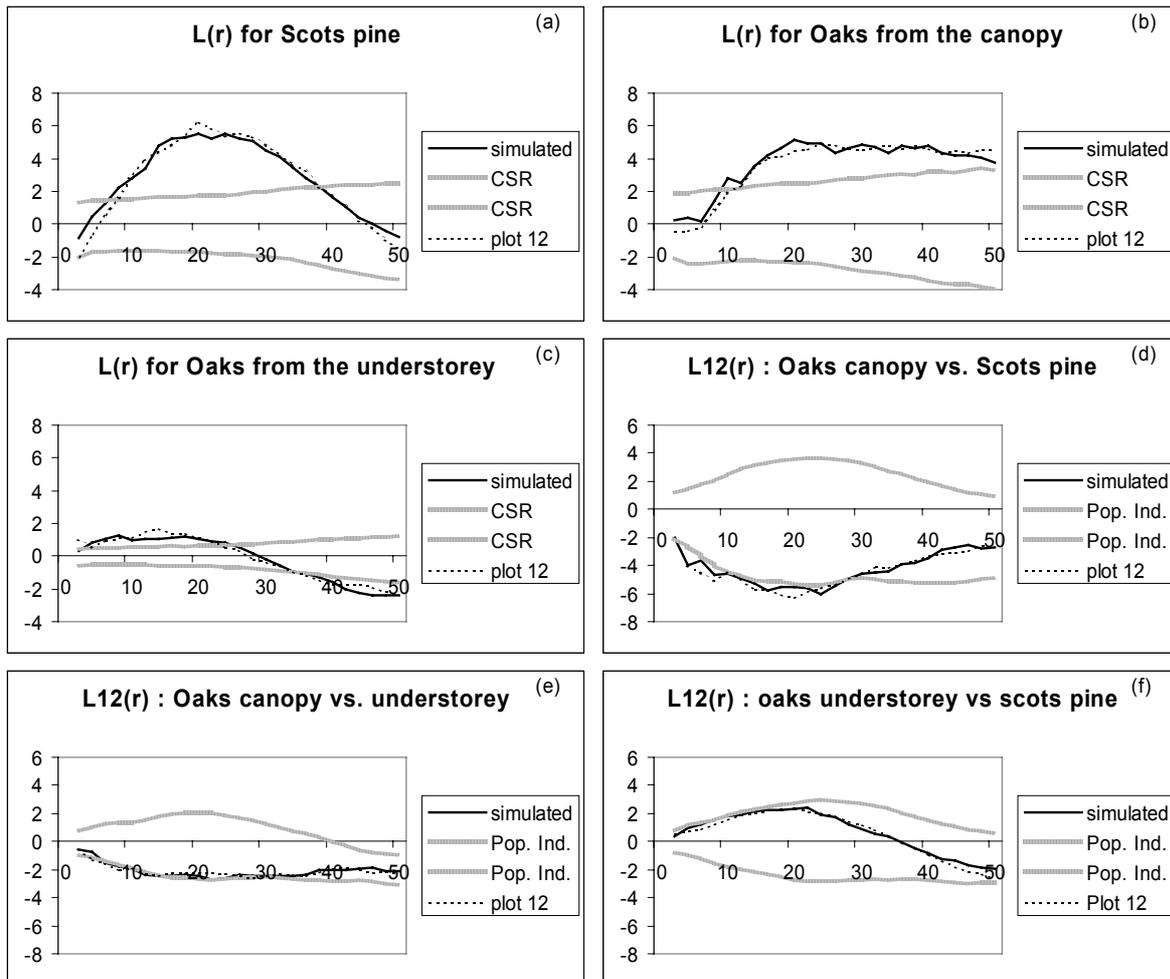
Figure 11 : Spatial structure analysis for the three sub-populations simulated with the mimetic process. *L(r)* function (black line), and the corresponding confidence interval for the C.S.R. null hypothesis (grey line), compared to the real values measured on plot 12 (dotted line) : (a) Scots pines, (b) oaks from the canopy, (c) oaks from the understorey. Intertype $L_{12}(r)$ function (black line), and the corresponding confidence interval for the Population Independence null hypothesis (grey line), compared to the real values measured on plot 12 (dotted line) : (d) oaks vs. Scots pines in the canopy, (e) oaks from the canopy vs. the understorey, and (f) oaks from the understorey vs. Scots pines.

5. Discussion

In this paper, we have proposed a mimetic point process, aimed at simulating spatial point patterns whose characteristics - and more precisely *L(r)* and $L_{12}(r)$ functions - are as close as possible to real values of these functions measured on an experimental plot. This mimetic point process is based on a generalised Gibbs process, whose global cost function directly depends on the difference between the real measured values of *L(r)* and $L_{12}(r)$, and the corresponding values computed on the simulated pattern at each iteration of the algorithm. We applied this point process to a mixed Oak - Scots pine forest, but the method described in this paper can of course be used in any other context where a simulation of realistic point patterns is needed. It can also be easily adapted to other characteristics of the spatial structure.

When applied to our experimental plot in a mixed forest stand composed of 3 sub-populations, the mimetic point process has indeed succeeded in simulating very realistic patterns, reproducing quite precisely the values of the three $L(r)$ functions of each sub-population, as well as the three $L_{12}(r)$ functions of each intertype interactions between sub-populations. The realisations we obtained were more realistic than previous realisations obtained more classically, by using *ad hoc* point processes with parameters either defined through expertise, or more precisely fitted on the real values of $L(r)$ and $L_{12}(r)$ functions. We therefore believe that this mimetic point process could improve the simulation of realistic patterns in ecology, which is usually done by expertise (Pretzsch 1997; de Coligny et al. 2004) or point pattern fitting (Rathbun & Cressie 1994; Batista & Maguire 1998). We think it will especially be of great help to facilitate the use of individual based models by forest managers. Indeed, these models require very detailed measures on the initial states, that are too expensive to be available in classical forest management. This mimetic point process could be used to simulate realistic virtual stands to be used as initial states for individual based models. It will only require to have enough data to estimate $L(r)$ and $L_{12}(r)$ functions, for instance on small experimental plots.

However, this method is a totally *ad hoc* simulation process, based on the minimisation of a global cost function, which does not allow to take ecological or mechanistic considerations into account. When simulating realistic patterns, ecologists can be interested in choosing the form of the simulated pattern in coherence with their knowledge on the ecosystem. For instance, Batista & Maguire (1998) are using Neyman-Scott processes because heavy seeds are clustered ; and Pretzsch (1997) is using inhomogeneous Poisson processes to simulate a repulsion between species in mixed stands. Such considerations are not possible with our mimetic process. Moreover, as we simulate the different sub-populations together, we can not take into account any hierarchy between sub-populations, which can be done when using hierarchical Gibbs processes (Högmander & Särkkä 1999). Lastly, we still need to define a proper validation for this point process, that would be based on independent data and on other characteristics than those that are used in the global cost function.

The simple depletion replacement algorithm we used in the mimetic process also has some limits. First, it is a simple optimisation algorithm, that does not insure a convergence towards the global minimum of the cost function, but only towards a local minimum. Loussier (2003) showed that this algorithm could be improved by using simulated annealing methods. However, what we simulate here is not exactly a Gibbs process, but only a very simple estimation, that does not simulate properly the variability of the process. Moreover, as the minimisation method does not take the statistical properties of the system into account, its validation will be difficult. Second, this algorithm is very time consuming. Thanks to the speed of recent computers, this is not really a problem for patterns with three sub-populations, but unfortunately it is far too slow when a great number of sub-populations have to be defined, which is the case for instance in tropical forest (Loussier, 2003). The algorithm should therefore still be improved, for instance by optimising the points to be deleted and replaced, instead of choosing them at random.

The definition of the global cost function, which is the optimisation criterion of the mimetic process, rises some essential questions. Indeed, different characteristics of the pattern can be used, and would lead to differences in the simulated patterns. First, we have to precise the ranges at which we want the spatial structure to be realistically simulated. If we use simple characteristics, such as Clark & Evans index, then the mimetic process will probably simulate realistically the short distance spatial structure only. In our application, we decided to use $L(r)$

and $L_{12}(r)$ functions, because we wanted to simulate a realistic spatial structure at different ranges r. Then we defined the maximum value of r in the global cost function as 50m, because we wanted the simulated $L(r)$ and $L_{12}(r)$ functions to be similar to the real ones up to 50m. Second, we have to choose the characteristics used in the global cost function. For instance, Batista & Maguire (1998), following Diggle (1983), used $K(r)$. Indeed, $K(r)$ has the advantage of having unbiased estimators, and of being directly linked to the number of neighbours. However, as the variance of the estimator of $K(r)$ increases with r, a simple least square criterion on $K(r)$ may lead to over estimate the influence of the large range spatial structure. Therefore, Diggle (1983) suggested to use a modified least square criterion with tuning variables. We preferred to use $L(r)$, whose variance is more stable, with a simple least square criterion. Other functions, such as $g(r)$, could be used and would lead to other results. Third, we have to choose whether to use some weighting factors. Weighting factors of different types in the global cost function give a potential fine tuning of the mimetic process (Loussier, 2003). It can be used to give more importance to some ranges (for instance short ranges, which may have more ecological meaning) compared to others. It also allows to give more importance to some sub-populations, for instance because they have an ecological interest, or because other sub-populations are not very well identified. Weighting factors could also be used to take into account not only the values of $L(r)$, but also its relative position to the confidence interval. Finally, we could also take some heterogeneity into account by using local values of the first order properties of the system as additional constraints.

The variability of the mimetic point process is also an important point. As we have seen, the algorithm used to simulate this mimetic point process converges towards a local, or a global minimum of the global cost. The number of iterations of this algorithm is thus an important parameter, that is linked to the intensity of the similarity between simulated and real $L(r)$ functions. The same problem occurs with classical Gibbs processes (Tomppo 1986). We therefore have to decide what we really want to simulate. If we want to obtain a realisation whose spatial structure characteristics are exactly the same than the measured ones - or at least as close as possible -, than we have to find the global minimum, which means a great number of iterations, and maybe an improvement of the algorithm. Note that this will not be a Gibbs process *sensu stricto*. However an exact similarity may not be that realistic. Indeed, in reality, the estimated spatial structure also varies from one realisation to another. Therefore, if we want to be realistic, we have to simulate spatial patterns with a certain variability. This could be obtained by adapting the number of iterations to the variability we want to reproduce. For instance, in our mixed forest stand, we will use other plots corresponding to the same type of stand and spatial structure, in order to estimate the variability of the $L(r)$ and $L_{12}(r)$ functions for this type of stand. We will then define the number of iterations in the mimetic process so that the variability of the simulated patterns corresponds to the real variability of the spatial structure in this forest type.

6. Acknowledgement

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