Reducing algorithm complexity in trees

Christophe Godin, Frédéric Boudon, Christophe Pradal
christophe.godin@inria.fr

FSPMs make intensive use of algorithms that manipulate the branching structure of plants. These algorithms may serve a variety of computational purposes related to these branching structures such as displaying them on screen, extracting data samples and statistics from them, comparing them, computing flows of matter (water, nutrients, signals) through them, computing the propagation of physical forces in them, or growing them. In general, the computational complexity of all these algorithms is directly proportional to a power (greater than 1) of the number \( N \) of components of the considered branching structure. In most FSPMs, \( N \) is relatively high and tends to grow exponentially with the plant’s age, at least in the first stages of plant development. For this reason, most FSPM algorithms are computationally expensive, with a time and memory complexity at least proportional to \( N \).

Different strategies have been considered to reduce this computational complexity and they most of them make use of a change of scale in the plant representation. Rather than computing light interception on each leaf of a tree for example, more efficient algorithms can be achieved by assessing the light intercepted by crownlets at a coarser scale. These approaches are computationally efficient but require both a change of plant representation and of the associated models.

In this paper, we present a new way of reducing algorithm complexity in FSPMs by compressing plant branching structures representation at a given scale. For this, we make use of the possibility to compress tree branching structures as directed acyclic graphs (DAGs) either in a lossless or approximate manner. We show how these DAG-transforms of branching systems can be used to reformulate a wide class of algorithms that operate on trees, and that, if \( N \) denotes the number of tree components and \( D \) the number of DAG components corresponding to the compressed tree, the algorithm complexity are in general reduced from \( O(N) \) to \( O(D) \) in both time and space. For a special class of trees, called self-nested trees, with high compression capacity, the complexity is reduced from \( O(N) \) to \( O(\log(N)) \). Together with the possibility to approximate any tree by a tree in the class of self-nested trees, this opens up the way to perform a wide range of FSPM algorithms on any tree with remarkable spatial and temporal efficiency.

We illustrate our approach on two markedly different types of algorithms for FSPMs. First, we show how the geometry of tree branching structures can be efficiently compressed and used. We then demonstrate the benefits of such an approach for the encoding of the geometric representation of various types of classical plant architectures. We then show how our approach can be used to achieve drastic simplification in the computation of flows in branching structures. Based on previous works that presented how flows can be recursively computed in trees, we show that compression techniques can be used to reduce substantially the complexity of flow computation in trees. We then use this new computational scheme to explore the impact of tree architecture on flow propagation in trees.